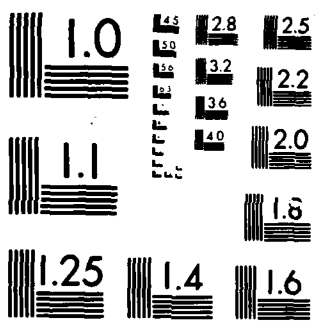
AD-A163 991 ATOMIC MOTION IN A STANDING WAVE(U) AIR FORCE INST OF TECH WRIGHT-PATTERSON AFB OH SCHOOL OF ENGINEERING J E SCOTT DEC 84 AFIT/GEP/ENP/84D-9 1/1

UNCLASSIFIED F/G 7/4 NL

END

FILMED



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS 1963-A

(1)

ATOMIC MOTION IN A STANDING WAVE

THESIS

Joseph E. Scott
Captain, USAF

AFIT/GEP/ENP/84D-9

DEPARTMENT OF THE AIR FORCE
AIR UNIVERSITY
AIR FORCE INSTITUTE OF TECHNOLOGY

Wright-Patterson Air Force Base, Ohio

AFIT/GEP/ENP/84D-9

ATOMIC MOTION IN A STANDING WAVE

THESIS

Joseph E. Scott
Captain, USAF

AFIT/GEP/ENP/84D-9

AFIT/GEP/ENP/84D-9

ATOMIC MOTION IN A STANDING WAVE

THESIS

Presented to the Faculty of the School of Engineering
of the Air Force Institute of Technology
Air University
In Partial Fulfillment of the
Requirements for the Degree of
Master of Science in Engineering Physics

Joseph E. Scott, B.S.
Captain, USAF

December 1984

Preface

This study of the atomic motion in a standing wave was motivated by the challenge presented in understanding the basic physical processes involved. The study of resonance radiation pressure, while dating back to the origins of quantum mechanics, is still a new and unexplored field. I am deeply indebted to my advisor, Major Richard J. Cook, for sparking my interest in this field, and for helping me to understand the nature of the investigation. I would also like to express my gratitude to all my friends in Dayton, Ohio, who helped me through the difficult times with their support and prayers.

Table of Contents

Page

Preface ii

List of Symbols. iv

List of Figures. vii

Abstract viii

I. Introduction 1

II. Background. 3

Radiation Force. 3
Atomic Cooling in Resonant Radiation 4
Neutral Atom Trapping. 5
Doppleron Resonances 5
Radiation Force Reversals. 6
Ehrenfest-Bloch Equations. 7
Weak Field Case. 9

III. Analytical Solution. 11

Fourier Series Solution. 11
Truncated Fourier Series 17

IV. Numerical Solution. 19

Constant Velocity Program. 19
Effect of u on the Radiation Force 22
Time-Averaged Force Reversals. 25
Comment on Constant Velocity Approximation . . . 26
Variable Velocity Program. 28
Fast Fourier Transform 30

V. Conclusions and Recommendations. 33

Appendix A: The Optical Bloch Equations 37

Appendix B: Computer Source Code. 45

Constant Velocity Program. 45
Variable Velocity Program. 47
Fast Fourier Transform Program 49

Bibliography 53

Vita 55

List of Symbols

A	Einstein A-Coefficient for spontaneous emission (sec^{-1})
c	Speed of Light (3×10^{10} cm/sec)
Δ	Laser frequency detuning ($\Delta = \omega - \omega_0$)
δ_{nm}	Kronecker delta
δp	Change in momentum within standing wave
δv	Change in velocity within standing wave
$E(t)$	Electric field of light wave
$\mathcal{E}_0$	Maximum amplitude of electric field
E_n	Eigenvalue for state n
F	Radiation or light pressure force (dynes)
$\overline{F}$	Time-averaged radiation force
$F_{\parallel}$	Longitudinal force component
$F_{\perp}$	Transverse force component
$F_{\parallel}^{0}, F_{\perp}^{0}$	DC components of Fourier series for force
$F_{\parallel,n}^{c}, F_{\parallel,n}^{s}, F_{\perp,n}^{c}, F_{\perp,n}^{s}$	Fourier coefficients for force

$\hat{H}$	Hamiltonian for atom
$\hat{H}_0$	Unperturbed Hamiltonian for atom
$\hat{H}'$	Perturbed Hamiltonian
$\hbar$	Planck's constant $/2\pi$ $(1.0546 \times 10^{-27}$ erg-sec)
i	$(-1)^{1/2}$
k	Wave propagation constant $= \frac{2\pi}{\lambda} = \frac{\omega}{c}$ (cm^{-1})
λ	Wavelength of standing wave (cm)
m	Mass of atom (grams)
$\hat{\mu}$	Atomic dipole moment operator
μ	Atomic dipole moment
$\Omega(t)$	Rabi flopping frequency (sec^{-1})
Ω_0	Maximum Rabi frequency $= \frac{\mu E_0}{\hbar}$ (measure of field strength)
ω	Laser frequency (sec^{-1})
ω_0, ω_{nm}	Atomic transition frequency $= \frac{E_n - E_m}{\hbar}$
P_n	Probability of being in state n
π	PI = 3.141592654
ψ_n	Wavefunction of state n

$\hat{\rho}, \rho_{nm}$	Density matrix
t	Time
τ	1/2 period of force
θ	Phase of light wave
u, v, w	Bloch variables
u_0, v_0, w_0	DC components of Fourier solution to Bloch equations
$\alpha, \beta, \gamma, \delta, \epsilon, \zeta$	Fourier coefficients for Bloch variables
$\vec{V}$	Bloch vector
v	Atomic velocity
$\pm v_0$	Zero force velocity points
x	Axis in direction of atomic motion and wave propagation

List of Figures

Figure		Page
1	Force as a function of atomic velocity for various field intensities [11]	7
2	Plots of: (a) field strength, (b), (c), and (d) Bloch Variables, and (e) radiation force (dynes) for constant velocity $v = 2 \times 10^3$ cm/sec	20
3	Plots of: (a) field strength, (b), (c), and (d) Bloch Variables, and (e) radiation force (dynes) for constant velocity $v = 6 \times 10^3$ cm/sec	21
4	Plot of u for various field strengths	23
5	Bloch variable u and force (dynes) for $\Omega_0 = 1A$ around a node in the standing wave. (Shaded area is the difference between the areas under the positive and negative force curves).	24
6	Bloch variable u and force (dynes) for $\Omega_0 = 5A$ around a node in the standing wave. (Shaded area is the difference between the areas under the positive and negative force curves).	24
7	Time-averaged force as a function of velocity for two field strengths	25
8	Time-averaged force for $v = 2 \times 10^3$ cm/sec as a function of field strength.	26
9	Comparason of time-averaged force calculated from variable velocity program and constant velocity program.	29
10	Descrete harmonic frequency spectrum for u calculated at: (a) $\Omega_0 = 1A$, (b) $\Omega_0 = 5A$, and (c) $\Omega_0 = 20A$	31
11	Descrete harmonic frequency spectrum for the radiation force calculated at: (a) $\Omega_0 = 1A$, (b) $\Omega_0 = 5A$, and (c) $\Omega_0 = 20A$	32

Abstract

The motion of an atom in an intense resonant standing laser wave is studied using the Ehrenfest-Bloch equations. Two approaches are taken to determine the radiation or light pressure force on the atom. The first, an attempt to find a Fourier series solution to the optical Bloch equations failed, but Fourier coefficient recursion formulas were found. The second, a numerical calculation of the Bloch variables and the radiation force was successful in obtaining detailed plots of the instantaneous values. The effect of assuming constant atomic velocity is studied, and Fast Fourier Transform descrete harmonic frequency spectra of one of the Bloch variables and the light pressure force are obtained. Finally, the Ehrenfest-Bloch equations are compared with other theories of resonant radiation pressure.

ATOMIC MOTION IN A STANDING WAVE

I. Introduction

With the advent of high power tunable laser systems, the study of the effect of radiation pressure on atoms near atomic resonant frequencies has developed into a fertile field of investigation. Of particular interest in recent years is the study of the motion of an atom in an intense standing laser wave. The light pressure of such standing waves may provide a means to damp out unwanted thermal motion in a beam of particles, or to cool and trap single neutral atoms within traps of laser light. With such setups, doppler-free spectroscopy could be performed on single trapped atoms. Single atom experimentation may become the norm of modern experimental physics. A trapped neutral atom could also be used as an oscillator to create a highly accurate atomic time base for physical measurements. The potential uses of light pressure to change the motion of an atom is a long and very imaginative list.

But, to be able to use the effects of radiation force to perform these investigations, it is necessary to characterize and understand what happens between the light wave and the atom. Several theorems of light pressure have been developed which describe interaction between the light wave and the atom. This thesis describes the use of the Ehrenfest-Bloch equations to characterize the atomic motion

in an intense standing laser wave.

In this work, an attempt was made to analytically solve the optical Bloch equations using a Fourier series solution. Although it was unsuccessful in obtaining a complete solution to the Bloch equations, a set of Fourier coefficient recursion formulas were found that can be used to study the coupling between terms in the series.

In addition to the analytical efforts, the Ehrenfest-Bloch equations were solved numerically to get a picture of the atomic motion as a function of time and field intensity. Detailed plots were created that describe the internal motion of the atom with the Bloch vector, and the external motion under the influence of the light pressure force. As a part of the study, the assumption of constant atomic velocity and its effect on the results of the numerical calculations was investigated. Also, a Fast Fourier Transform was performed on the Bloch variable $\mathcal{U}$, and the light pressure force to determine their dependence on higher harmonic frequencies as a function of field intensity.

The results obtained from using the Ehrenfest-Bloch equations are comparable to the published results obtained through other theories of the light pressure forces. But, by using the Ehrenfest-Bloch equations, one obtains a detailed instantaneous view of the atom as it moves through the standing wave. This view is a significant step toward the complete understanding of the interaction between light and matter.

II. BACKGROUND

Radiation Force

Laser light tuned to a frequency near the resonant transition frequency of an atom is strongly scattered by the atom. In the scattering process, momentum is transfered from the field of the light to the atom. The rate of this momentum transfer is called radiation force or light pressure on the atom. Light pressure is a composition of two forces on the atom: the spontaneous or longitudinal force, and the dipole or gradient force.

The spontaneous force results from the absorption of the light wave which excites the atom, and the subsequent decay of the atom to the ground state. Because the spontaneous emission occurs in a completely random direction, the atom feels no net change in momentum due to the emission. The momentum change is due to the absorption of the light wave and thus occurs in the direction of light propagation, hence the name, longitudinal force.

The dipole force, on the other hand, is a result of the interaction between the induced atomic dipole moment and the amplitude gradient of the electric field of the light wave. This force is characterized by its direction toward or awa from increasing field strength depending on whether the laser is tuned to a frequency below or above the resonant transition frequency of the atom. When the laser is tuned exactly on resonance, the atom experiences no dipole force.

Because the of the profile of a laser beam, this force is usually directed perpendicular to the direction of light propagation. Thus, the dipole force is often called the transverse force.

Atomic Cooling in Resonant Radiation

The study of atomic motion in resonant or near-resonant radiation has received considerable interest in recent years since the proposal by T.W. Hänsch and A.L. Schawlow to use light pressure force to cool atoms by placing them within a standing light wave with a frequency tuned slightly below the resonant transition frequency of an atom [6]. In their proposal, the thermal motion of the atoms will cause the frequency of the light to be doppler shifted such that one of the two counter-propagating waves of the standing wave closer to resonance, while the other would be moved farther from resonance. This would cause the atoms to absorb more momentum from the wave closer to resonance, and thus experience the spontaneous force opposing their motion. In this way, the motion of the atoms in the standing wave would be damped out and the atoms cooled.

This relatively simple idea of applying light pressure to cool atoms led to the development of detailed theories of the light pressure force [3], [5], [7], [10], [13]. These theories characterized the nature of the light pressure forces and found them to be more complex than first envisioned by Hänsch and Schawlow.

Neutral Atom Trapping

As the theories of light pressure were being developed, it became apparent that it was theoretically possible to use radiation pressure to create a potential well into which an atom could be placed. The idea of a "light trap" to hold an extremely cold atom within a beam of light spurred interest in the study of the motion of an atom in an intense standing laser wave [2], [9]. As discussed in the introduction, the successful cooling and trapping of a neutral atom would revolutionize experimental physics, and indeed, is one of the most ambitious goals of modern physics today.

Doppleron Resonances

In the development one theory of the light pressure in a standing wave, E. Kyrölä and S. Stenholm predicted the existence of multiphoton or Doppleron resonances to occur at when the atomic velocity was

$$v = \pm \frac{|\Delta|}{k(2n+1)} \tag{1}$$

This was described as the result of absorbing $n+1$ photons from one of the two counter-propagating light waves that make up the standing wave, and emitting n photons into the other. The name, Doppleron, was applied because the process is caused by the Doppler shift of the field for the moving atom. These resonances were found to become significant for large field strengths.

Radiation Force Reversals

Calculations from a theory of resonance light pressure by V.G. Minogin and O.T. Serimaa showed that the Doppleron resonances predicted by Kyrölä and Stenholm appeared in the form of variation in the light pressure force [11].

They presented the light pressure force as the sum of two orthogonal components: the longitudinal force $F_{\parallel}$, and the transverse or gradient force $F_{\perp}$. These components were expressed in the form of Fourier expansions:

$$F_{\parallel} = F_{\parallel}^{0} + \sum_{n=1}^{\infty} \left(F_{\parallel,n}^{c} \cos 2nkx + F_{\parallel,n}^{s} \sin 2nkx \right) \tag{2a}$$

$$F_{\perp} = F_{\perp}^{0} + \sum_{n=1}^{\infty} \left(F_{\perp,n}^{c} \cos 2nkx + F_{\perp,n}^{s} \sin 2nkx \right) \tag{2b}$$

where the coefficients are a rather complicated set of convergent partial fractions. For very high intensities and and low atomic velocities, the force actually changes sign. Instead of being a damping or cooling force, the light pressure force becomes an accelerating or heating force.

The force resonances found by Minogin and Serimaa are centered approximately at the velocities predicted in equation (1), shifted toward greater velocities and power broadened as the intensity of the radiation field increases.

Figure (1) is a plot of the time averaged longitudinal force calculated from the expansion of the Fourier coefficients.

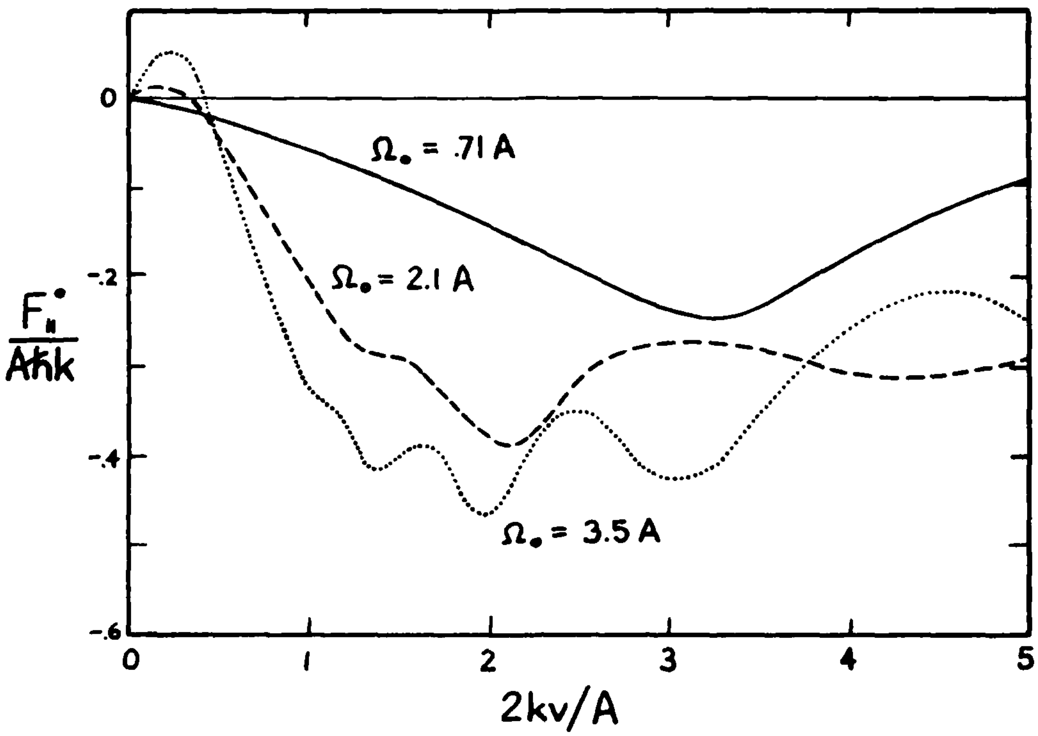


Figure 1. Force as a function of atomic velocity for various field intensities [11].

Ehrenfest-Bloch Equations

The difficulty of working with the statement of the light pressure force developed by Minogin and Serimaa lies in the complexity of the partial fraction solution for the Fourier coefficients of the force.

A simplified theory of near resonance light pressure was developed by R.J. Cook [3]. This theory, based on Ehrenfest's Theorem and the optical Bloch equations provides

a simple statement of the radiation force through a system of four coupled equations called the Ehrenfest-Bloch equations

$$\dot{u} = -\tfrac{1}{2}Au + \Delta v \tag{3a}$$

$$\dot{v} = -\Delta u - \tfrac{1}{2}Av + \Omega w \tag{3b}$$

$$\dot{w} = -\Omega v - A(w+1) \tag{3c}$$

and

$$\vec{F} = \tfrac{1}{2}\hbar\left(u\nabla\Omega + v\Omega\nabla\theta\right) \tag{4}$$

Equations (3) are the optical Bloch equations for a two-level atom in the Rotating Wave Approximation (Appendix A), and Equation (4) is the equation of motion of the atom obtained from Ehrenfest's Theorem.

For the case of a standing wave, the electric field is a function of position, $E(x) = \mathcal{E}_0 \cos kx$, and the phase is given by $\theta = 0$. Therefore, the Rabi frequency becomes $\Omega = \Omega_0 \cos kx$ where $\Omega_0 = \mu\mathcal{E}_0/\hbar$. The radiation force can then be written as

$$F = -\tfrac{1}{2}\hbar k\,\Omega_0 u \sin kx \tag{5}$$

If the velocity, V , of the atom remains constant (a reasonable approximation), then $x = vt$ and the Ehrenfest-Bloch equations become

$$\dot{u} = -\tfrac{1}{2}Au + \Delta v \tag{6a}$$

$$\dot{v} = -\Delta u - \tfrac{1}{2}Av + \Omega_0 \cos kvt \; w \tag{6b}$$

$$\dot{w} = -\Omega_0 \cos kvt \; v - A(w+1) \tag{6c}$$

and

$$F = -\tfrac{1}{2}\hbar k \Omega_0 \, u \sin kvt \tag{7}$$

The light pressure force is determined by solving the Bloch equations and substituting the value of u into equation (7).

The Weak Field Case

The Ehrenfest-Bloch equations can be solved analytically for the case of the weak field excitation [11]. In this case, the atom will remain near the ground state ($w \approx -1$) and the Bloch equations can be written as

$$\dot{u} = -\tfrac{1}{2}Au + \Delta v \tag{8a}$$

$$\dot{v} = -\Delta u - \tfrac{1}{2}Av - \Omega_0 \cos kvt \tag{8b}$$

$$\dot{w} = 0 \tag{8c}$$

The solution for this set of equations was found to be

$$u = \alpha \cos kvt + \beta \sin kvt \tag{9a}$$

$$v = \gamma \cos kvt + \delta \sin kvt \tag{9b}$$

$$w = -1 \tag{9c}$$

where the coefficients can be found by solving

$$\alpha = \frac{\frac{1}{2}A\beta - \Delta\delta}{kv} \tag{10a}$$

$$\beta = \frac{-\frac{1}{2}A\alpha + \Delta\gamma}{kv} \tag{10b}$$

$$\gamma = \frac{\Delta\beta + \frac{1}{2}A\delta}{kv} \tag{10c}$$

$$\delta = \frac{-\Delta\alpha - \frac{1}{2}A\gamma - \Omega_o}{kv} \tag{10d}$$

The force is given by substituting the solution for u into equation (7) to get

$$F = -\tfrac{1}{2}\Omega_o \hbar k \left(\alpha \cos kvt + \beta \sin kvt\right) \sin kvt \tag{11}$$

Or, when the force is averaged over time,

$$\overline{F} = -\tfrac{1}{4}\Omega_o \hbar k \beta \tag{12}$$

where

$$\beta = \frac{-\Delta\Omega_o A kv}{\left[\Delta^2 - (kv)^2\right]^2 + (\frac{1}{2}A)^2\left[(\frac{1}{2}A)^2 + 2\Delta^2 + 2(kv)^2\right]} \tag{13}$$

This shows that in the weak field case, the direction of the force depends on the product, ΔV. A negative force opposes the motion of the atom and will slow it down, while a positive one is in the direction for the atomic motion and will speed it up. This is in agreement with the weak field results by Minogin and Serimaa.

III Analytical Solution

The first part of the thesis research was spent analytically manipulating the optical Bloch equations to see if they could be solved with a Fourier series. This series was then to be investigated to determine the dependence of the time-averaged force on the magnitude of the electric field of the standing laser wave.

This effort was unsuccessful in obtaining a Fourier series solution to the Bloch equations. Several recursion formulas, however, were found for the coefficients of the Fourier series. This section describes the Fourier coefficients and their use in approximating the weak field solution.

Fourier Series Solution

The weak field solution was found by approximating the optical Bloch equations with the assumption that the atom remained near the ground state. While this may be true, no information about the strength of such a weak field can be determined from the initial assumptions. Using the Fourier solution to solve the complete Bloch equations, the weak field case can be investigated by studying this solution

From equations (9), the complete solution of the Bloch equations could be assumed to be a Fourier series in which the higher order coefficients become negligible in the weak field case. Such a solution would be

$$u = u_0 + \sum_{n=1}^{\infty}\left(\alpha_n \cos nkvt + \beta_n \sin nkvt\right) \tag{14a}$$

$$v = v_0 + \sum_{n=1}^{\infty}\left(\gamma_n \cos nkvt + \delta_n \sin nkvt\right) \tag{14b}$$

$$w = w_0 + \sum_{n=1}^{\infty}\left(\epsilon_n \cos nkvt + \zeta_n \sin nkvt\right) \tag{14c}$$

where the coefficients would have to be determined to obtain the complete solution. By substituting the assumed solution into equations (6), one obtains

$$\begin{aligned} kv\sum_{n=1}^{\infty}\left(-n\alpha_n \sin nkvt + n\beta_n \cos nkvt\right) &= -\tfrac{1}{2}A\left[u_0 + \sum_{n=1}^{\infty}\left(\alpha_n \cos nkvt + \beta_n \sin nkvt\right)\right] \\ &\quad + \Delta\left[v_0 + \sum_{n=1}^{\infty}\left(\gamma_n \cos nkvt + \delta_n \sin nkvt\right)\right] \end{aligned} \tag{15a}$$

$$\begin{aligned} kv\sum_{n=1}^{\infty}\left(-n\gamma_n \sin nkvt + n\delta_n \cos nkvt\right) &= -\Delta\left[u_0 + \sum_{n=1}^{\infty}\left(\alpha_n \cos nkvt + \beta_n \sin nkvt\right)\right] \\ &\quad -\tfrac{1}{2}A\left[v_0 + \sum_{n=1}^{\infty}\left(\gamma_n \cos nkvt + \delta_n \sin nkvt\right)\right] \\ &\quad + \Omega_0 \cos kvt\left[w_0 + \sum_{n=1}^{\infty}\left(\epsilon_n \cos nkvt + \zeta_n \sin nkvt\right)\right] \end{aligned} \tag{15b}$$

$$\begin{aligned} kv\sum_{n=1}^{\infty}\left(-n\epsilon_n \sin nkvt + n\zeta_n \cos nkvt\right) &= -\Omega_0 \cos kvt\left[v_0 + \sum_{n=1}^{\infty}\left(\gamma_n \cos nkvt + \delta_n \sin nkvt\right)\right] \\ &\quad -A\left[w_0 + \sum_{n=1}^{\infty}\left(\epsilon_n \cos nkvt + \zeta_n \sin nkvt\right)\right] \\ &\quad -A \end{aligned} \tag{15c}$$

With rearranging, equation (15a) becomes

$$\sum_{n=1}^{\infty} -n\alpha_n \sin nkvt + \sum_{n=1}^{\infty} n\beta_n \cos nkvt$$

$$= \left[\frac{-\frac{1}{2}Au_o + \Delta v_o}{kv}\right]$$

$$+\sum_{n=1}^{\infty}\left[\frac{-\frac{1}{2}A\alpha_n + \Delta\gamma_n}{kv}\right]\cos nkvt$$

$$+\sum_{n=1}^{\infty}\left[\frac{-\frac{1}{2}A\beta_n + \Delta\delta_n}{kv}\right]\sin nkvt \qquad (16)$$

Now, equating the coefficients, one obtains

$$\boxed{\begin{aligned} 0 &= \frac{-\frac{1}{2}Au_o + \Delta v_o}{kv} \\ \alpha_n &= \frac{\frac{1}{2}A\beta_n - \Delta\delta_n}{kv} \\ \beta_n &= \frac{-\frac{1}{2}A\alpha_n + \Delta\gamma_n}{kv} \end{aligned}} \qquad \begin{aligned} (17) \\ (18) \\ (19) \end{aligned}$$

Similarly, equation (15b) can be rearranged

$$\sum_{n=1}^{\infty} -n\gamma_n \sin nkvt + \sum_{n=1}^{\infty} n\delta_n \cos nkvt$$

$$= \left[\frac{-\Delta u_o - \frac{1}{2}Av_o}{kv}\right] + \frac{\Omega_o w_o}{kv}\cos kvt$$

$$+\sum_{n=1}^{\infty}\left[\frac{-\Delta\alpha_n - \frac{1}{2}A\gamma_n}{kv}\right]\cos nkvt$$

$$+\sum_{n=1}^{\infty}\left[\frac{-\Delta\beta_n - \frac{1}{2}A\delta_n}{kv}\right]\sin nkvt$$

$$+\sum_{n=1}^{\infty}\frac{\Omega_o\epsilon_n}{kv}\cos kvt \cos nkvt$$

$$+\sum_{n=1}^{\infty}\frac{\Omega_o\zeta_n}{kv}\cos kvt \sin nkvt \qquad (20)$$

Using the trigonometric identities,

$$\cos\theta\cos n\theta = \tfrac{1}{2}\cos(n+1)\theta + \tfrac{1}{2}\cos(n-1)\theta \tag{21a}$$

$$\cos\theta\sin n\theta = \tfrac{1}{2}\sin(n+1)\theta + \tfrac{1}{2}\sin(n-1)\theta \tag{21b}$$

equation (20) becomes

$$\begin{aligned}
&\sum_{n=1}^{\infty} -n\gamma_n \sin nkvt + \sum_{n=1}^{\infty} n\delta_n \cos nkvt \\
&\quad = \left[\frac{-\Delta u_0 - \frac{1}{2}A v_0}{kv}\right] + \frac{\Omega_0 w_0}{kv}\cos kvt \\
&\qquad + \sum_{n=1}^{\infty}\left[\frac{-\Delta\alpha_n - \frac{1}{2}A\gamma_n}{kv}\right]\cos nkvt \\
&\qquad + \sum_{n=1}^{\infty}\left[\frac{-\Delta\beta_n - \frac{1}{2}A\delta_n}{kv}\right]\sin nkvt \\
&\qquad + \sum_{n=1}^{\infty}\frac{\Omega_0\epsilon_n}{kv}\left[\tfrac{1}{2}\cos(n+1)kvt + \tfrac{1}{2}\cos(n-1)kvt\right] \\
&\qquad + \sum_{n=1}^{\infty}\frac{\Omega_0\xi_n}{kv}\left[\tfrac{1}{2}\sin(n+1)kvt + \tfrac{1}{2}\sin(n-1)kvt\right]
\end{aligned} \tag{22}$$

The summation indices in the last two terms in equation (22) can be changed to read

$$\begin{aligned}
&\sum_{n=1}^{\infty}\frac{\Omega_0\epsilon_n}{kv}\left[\tfrac{1}{2}\cos(n+1)kvt + \tfrac{1}{2}\cos(n-1)kvt\right] \\
&\quad = \sum_{m=2}^{\infty}\frac{\frac{1}{2}\Omega_0\epsilon_{m-1}}{kv}\cos mkvt + \sum_{m'=0}^{\infty}\frac{\frac{1}{2}\Omega_0\epsilon_{m'+1}}{kv}\cos m'kvt
\end{aligned} \tag{23}$$

$$\begin{aligned}
&= \sum_{m=2}^{\infty}\frac{\frac{1}{2}\Omega_0\epsilon_{m-1}}{kv}\cos mkvt + \frac{\frac{1}{2}\Omega_0\epsilon_1}{kv} \\
&\quad + \frac{\frac{1}{2}\Omega_0\epsilon_2}{kv}\cos kvt + \sum_{m'=2}^{\infty}\frac{\frac{1}{2}\Omega_0\epsilon_{m'+1}}{kv}\cos m'kvt
\end{aligned} \tag{24}$$

$$\begin{aligned}
&= \frac{\frac{1}{2}\Omega_0\epsilon_1}{kv} + \frac{\frac{1}{2}\Omega_0\epsilon_2}{kv}\cos kvt \\
&\quad + \sum_{m=2}^{\infty}\frac{\frac{1}{2}\Omega_0}{kv}\left(\epsilon_{m-1} + \epsilon_{m+1}\right)\cos mkvt
\end{aligned} \tag{25}$$

and

$$\sum_{n=1}^{\infty} \frac{\Omega_0 \zeta_n}{kv} \left[\tfrac{1}{2} \sin(n+1)kvt + \tfrac{1}{2} \sin(n-1)kvt \right]$$

$$= \frac{\tfrac{1}{2}\Omega_0 \zeta}{kv} \sin kvt + \sum_{m=2}^{\infty} \frac{\tfrac{1}{2}\Omega_0}{kv} (\zeta_{m-1} + \zeta_{m+1}) \sin mkvt \qquad (26)$$

Equation (22) thus becomes

$$-\gamma_1 \sin kvt + \sum_{n=2}^{\infty} -n\gamma_n \sin nkvt$$

$$+\delta_1 \cos kvt + \sum_{n=2}^{\infty} n\delta_n \cos nkvt$$

$$= \left[\frac{-\Delta u_0 - \tfrac{1}{2}Av_0 + \tfrac{1}{2}\Omega_0 \epsilon_1}{kv} \right]$$

$$+ \left[\frac{-\Delta\alpha_1 - \tfrac{1}{2}A\gamma_1 + \Omega_0 w_0 + \tfrac{1}{2}\Omega_0 \epsilon_2}{kv} \right] \cos kvt$$

$$+ \sum_{n=2}^{\infty} \left[\frac{-\Delta\alpha_n - \tfrac{1}{2}A\gamma_n + \tfrac{1}{2}\Omega_0(\epsilon_{n-1} + \epsilon_{n+1})}{kv} \right] \cos nkvt$$

$$+ \left[\frac{-\Delta\beta_1 - \tfrac{1}{2}A\delta_1 + \tfrac{1}{2}\Omega_0 \zeta_2}{kv} \right] \sin kvt$$

$$+ \sum_{n=2}^{\infty} \left[\frac{-\Delta\beta_n - \tfrac{1}{2}A\delta_n + \tfrac{1}{2}\Omega_0(\zeta_{n-1} + \zeta_{n+1})}{kv} \right] \sin nkvt \qquad (27)$$

And by, equating coefficients,

$$0 = \frac{-\Delta u_0 - \tfrac{1}{2}Av_0 + \tfrac{1}{2}\Omega_0 \epsilon_1}{kv} \qquad (28)$$

$$\gamma_1 = \frac{\Delta\beta_1 + \tfrac{1}{2}A\delta_1 - \tfrac{1}{2}\Omega_0 \zeta_2}{kv} \qquad (29)$$

$$\delta_1 = \frac{-\Delta\alpha_1 - \tfrac{1}{2}A\gamma_1 + \Omega_0 w_0 + \tfrac{1}{2}\Omega_0 \epsilon_2}{kv} \qquad (30)$$

and for $n \geq 2$,

$$\boxed{\begin{aligned} \gamma_n &= \frac{\Delta\beta_n + \frac{1}{2}A\delta_n - \frac{1}{2}\Omega_o(\zeta_{n-1} + \zeta_{n+1})}{nkv} \qquad (31) \\ \delta_n &= \frac{-\Delta\alpha_n - \frac{1}{2}A\gamma_n + \frac{1}{2}\Omega_o(\epsilon_{n-1} + \epsilon_{n+1})}{nkv} \qquad (32) \end{aligned}}$$

Similarly, equation (15c) can be rewritten as

$$\begin{aligned} &-\epsilon_1 \sin kvt + \sum_{n=2}^{\infty} -n\epsilon_n \sin nkvt \\ &+ \zeta_1 \cos kvt + \sum_{n=2}^{\infty} n\zeta_n \cos nkvt \\ &\quad = \left[\frac{-Aw_o - A - \frac{1}{2}\Omega_o\gamma_1}{kv}\right] \\ &\qquad + \left[\frac{-\Omega_o v_o - \frac{1}{2}\Omega_o\gamma_2 - A\epsilon_1}{kv}\right]\cos kvt \\ &\qquad + \sum_{n=2}^{\infty}\left[\frac{-A\epsilon_n - \frac{1}{2}\Omega_o(\gamma_{n-1} + \gamma_{n+1})}{kv}\right]\cos nkvt \\ &\qquad + \left[\frac{-A\zeta_1 - \frac{1}{2}\Omega_o\delta_2}{kv}\right]\sin kvt \\ &\qquad + \sum_{n=2}^{\infty}\left[\frac{-A\zeta_n - \frac{1}{2}\Omega_o(\delta_{n-1} + \delta_{n+1})}{kv}\right]\sin nkvt \end{aligned} \qquad (33)$$

And from equating coefficients,

$$\boxed{\begin{aligned} 0 &= \frac{-Aw_o - A - \frac{1}{2}\Omega_o\gamma_1}{kv} \qquad (34) \\ \epsilon_1 &= \frac{A\zeta_1 + \frac{1}{2}\Omega_o\delta_2}{kv} \qquad (35) \\ \zeta_1 &= \frac{-\Omega_o v_o - \frac{1}{2}\Omega_o\gamma_2 - A\epsilon_1}{kv} \qquad (36) \end{aligned}}$$

and for $n \geq 2$,

$$\boxed{\begin{aligned} \epsilon_n &= \frac{A\zeta_n + \frac{1}{2}\Omega_o(\delta_{n-1} + \delta_{n+1})}{kv} && (37) \\ \zeta_n &= \frac{-A\epsilon_n - \frac{1}{2}\Omega_o(\gamma_{n-1} + \gamma_{n+1})}{kv} && (38) \end{aligned}}$$

Equations (17) - (19), (28) - (32), and (34) - (38) form a coupled system of recursion formulas for determining the Fourier coefficients for the complete solution to the Bloch equations. While they were not solved for each coefficient during this study, they are still useful to provide some insight into the nature of the Fourier solution.

The Truncated Fourier Series

For the weak field case, the Bloch equations were approximated according to the assumption that the atom would remain in or near the ground state. Equations (9) list the analytical solutions in the weak field case.

If the recursion formulas obtained for the complete Bloch equations depict the coefficients of the Fourier series solution, then they would have to be the same as (9) in the limit of a weak field. Assuming that the coefficients of the higher harmonic terms are negligible in the weak field case, the coefficients can be written

$$\alpha = \frac{\frac{1}{2}A\beta - \Delta\delta}{kv} \qquad (39a)$$

$$\beta = \frac{-\frac{1}{2}A\alpha + \Delta\gamma}{kv} \qquad (39b)$$

$$\gamma = \frac{\Delta\beta + \frac{1}{2}A\delta}{kv} \qquad (39c)$$

$$\delta = \frac{-\Delta\alpha - \frac{1}{2}A\gamma + \Omega_o w_o}{kv} \qquad (39d)$$

$$\epsilon = \frac{A\zeta + \frac{1}{2}\Omega_o\delta}{kv} \qquad (39e)$$

$$\zeta = \frac{-\Omega_o v_o - A\epsilon}{kv} \qquad (39f)$$

and

$$0 = -\frac{1}{2}Au_o + \Delta v_o \qquad (40)$$

$$0 = \frac{-\Delta u_o - \frac{1}{2}Av_o + \frac{1}{2}\Omega_o\epsilon}{kv} \qquad (41)$$

$$0 = \frac{-Aw_o - A - \frac{1}{2}\Omega_o\gamma}{kv} \qquad (42)$$

Solving equation (42) for w_o and substituting into equation (39d), one obtains

$$\delta = \frac{-\Delta\alpha - \left(\frac{1}{2}A + \frac{\Omega_o^2}{2A}\right)\gamma - \Omega_o}{kv} \qquad (43)$$

It is immediately apparent that equations (39a), (39b), (39c), and (43) would be the same as (10) if

$$\frac{\Omega_o^2}{2A} \ll 1 \qquad (44)$$

which is a weak field indeed.

IV. Numerical Solution

The second area of effort in this thesis was to use the Ehrenfest-Bloch equations to determine the instantaneous motion of the atom as it moves through the standing wave. The computer programs that were used were written in Fortran 77 and can be found in Appendix B.

Because of the way the Ehrenfest-Bloch equations were derived for the case of a strong standing wave, two methods were used to program the equations. The first method kept the atomic velocity constant as in equations (6) and (7). The second method allowed the velocity to vary as it would in an actual experiment. Both methods provided useful information about the atomic motion in a standing wave.

Constant Velocity Program

The Ehrenfest-Bloch equations were first programmed with the assumption that the atomic velocity was constant. The values used for the Einstein A-Coefficient and the atomic transition frequency were $A = 10^{8}\ sec^{-1}$ and $\omega_0 = 10^{14}\ sec^{-1}$ respectively. These values were chosen arbitrarily and are not actual values for real atoms. The detuning was chosen to place the laser frequency one A-coefficient below the atomic transition frequency.

The Bloch equations were programmed exactly as they appear in equation (6). The derivatives of the variables were calculated at the beginning of each time step, then the

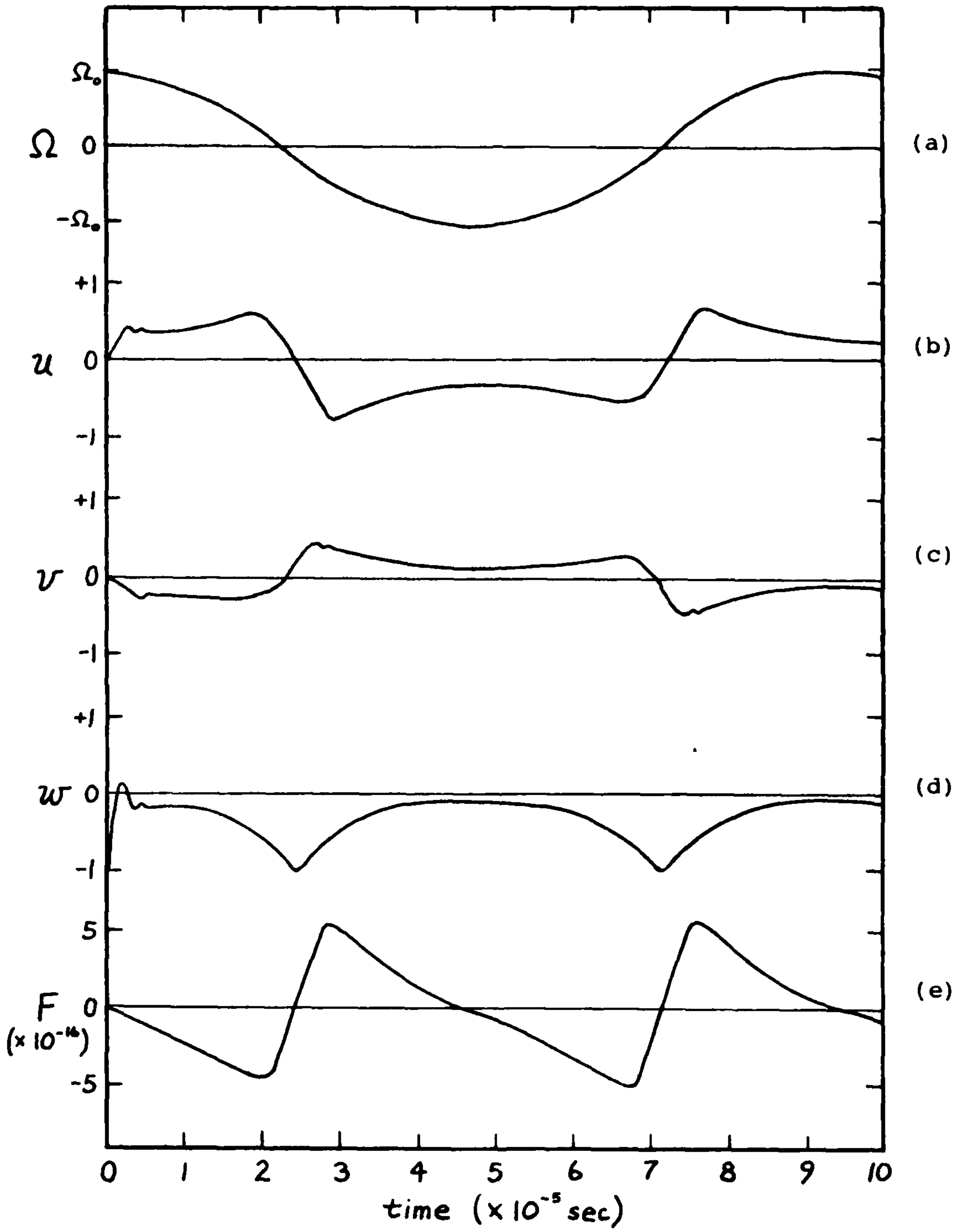


Figure 2. Plots of: (a) field strength, (b), (c), and (d) Bloch variables, and (e) radiation force (dynes) for constant velocity $v = 2 \times 10^3$ cm/sec .

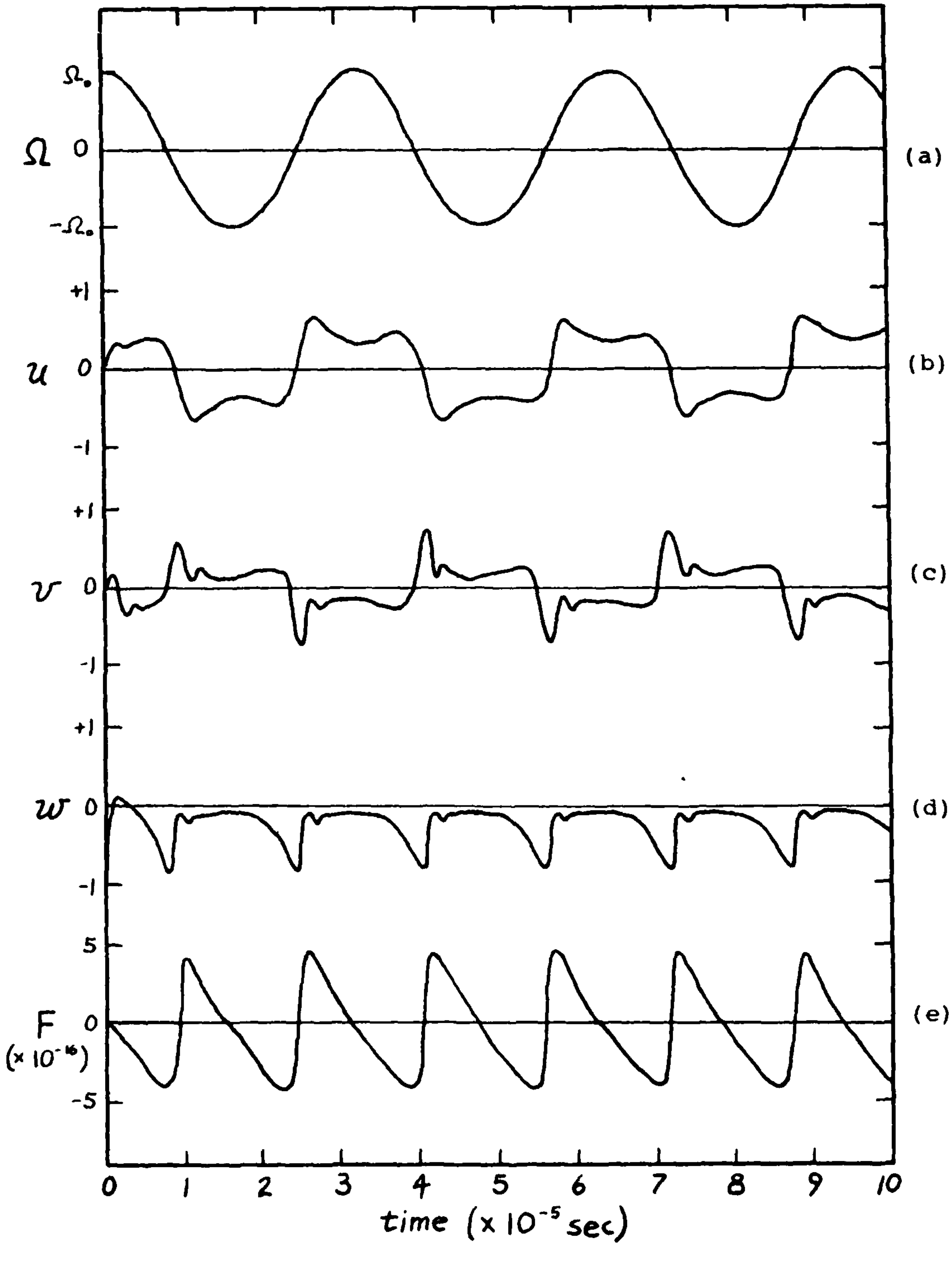


Figure 3. Plots of: (a) field strength, (b), (c), and (d) Bloch variables, and (e) radiation force (dynes) for constant velocity $v = 6\times10^3$ cm/sec .

time was incremented and the new values for the variables were found by a linear calculation across the time step. The size of the time increment that was used was 10^{-11} sec .

Figures (2) and (3) are pl s of the Bloch variables and the radiation force that were calculated for $\Omega_0 = 5A$ with the constant velocities, $v = 2\times10^3$ cm/sec and 6×10^3 cm/sec respectively. From these plots, it can be seen that the largest contribution to the light pressure occurs before and after the atom passes through a node in the standing wave. This is because the dipole force depends on the gradient of the field amplitude which is the greatest at the nodes. Close observation of Figures (2e) and (3e) show that the light pressure force for these velocities and field strength will not average to zero. In fact, the average force from Figure (2e) is a small positive quantity, while the average force from (3e) is a small negative quantity.

It can also be noted that the frequency of u and v is equal to the frequency of the standing wave as it is experienced by the moving atom, or the quantity, kv . The force and the inversion seem to have frequencies of twice this frequency or $2kv$. And finally, Rabi oscillations are observable in Figure (3) for v and w as a ringing in the Bloch variables.

Effect of u on the Radiation Force

In equation (7), the magnitude of the light pressure force for the case of an atom moving through a standing wave depends on the amplitude of the standing wave explicitly

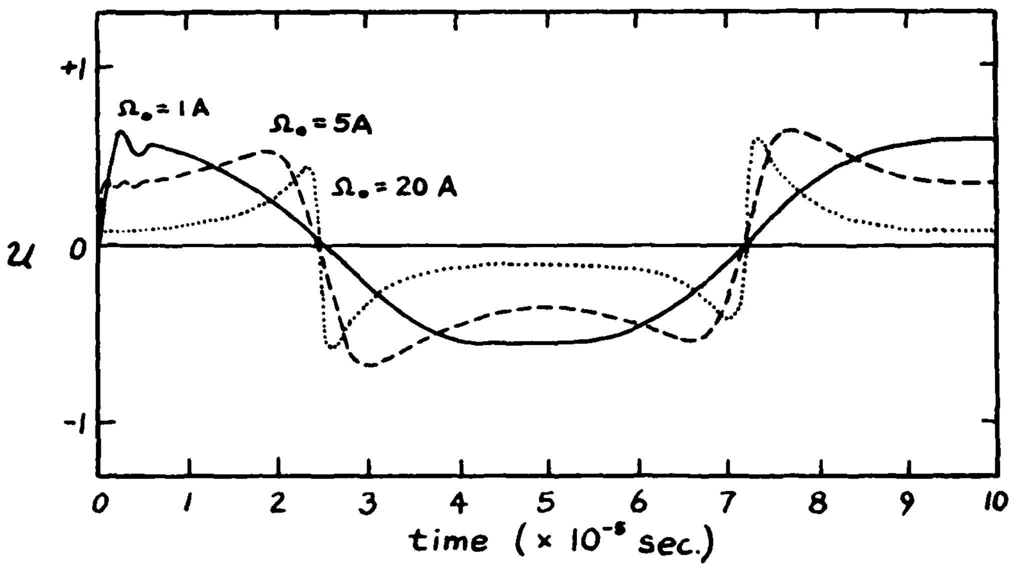


Figure 4. Plot of u for various field strengths.

through the Rabi flopping frequency, $\Omega_o = \mu \mathcal{E}_o/\hbar$, and implicitly through the Bloch variable u .

Figure (4) is a plot of u for three different field strengths. From this plot, it is found that u is clamped by the field of the standing wave, and that as the amplitude of the wave increases, u is clamped down to zero. It can also be seen that at the nodes of the standing wave, $|u|$ relaxes to a larger value. Upon closer observation, the tail immediately following the standing wave node is found to be larger than the one preceding it. Figures (2) and (3) show that the radiation force is negative preceding each node, and positive immediately following them.

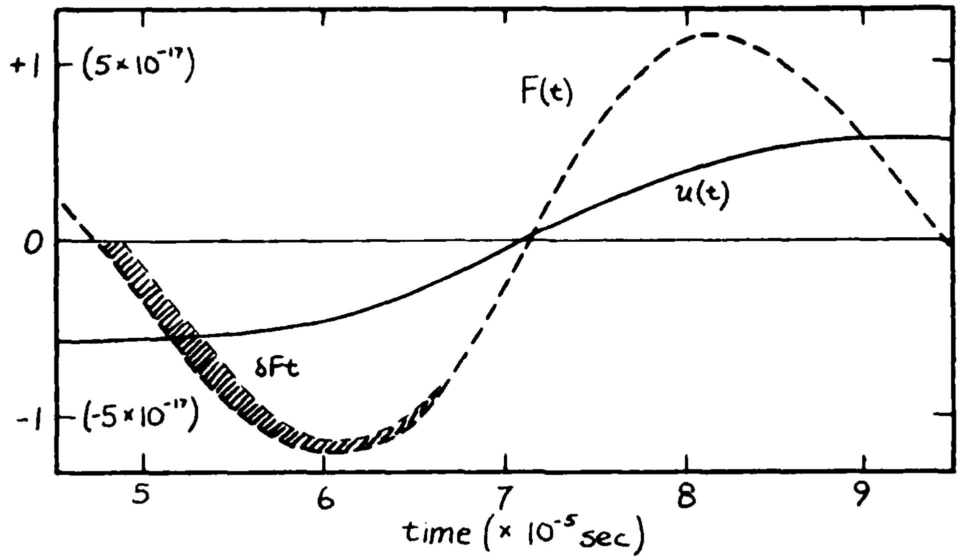


Figure 5. Bloch variable u and force (dynes) for $\Omega_o = 1A$ around a node in the standing wave. (Shaded area is the difference between the areas under the positive and negative force curves).

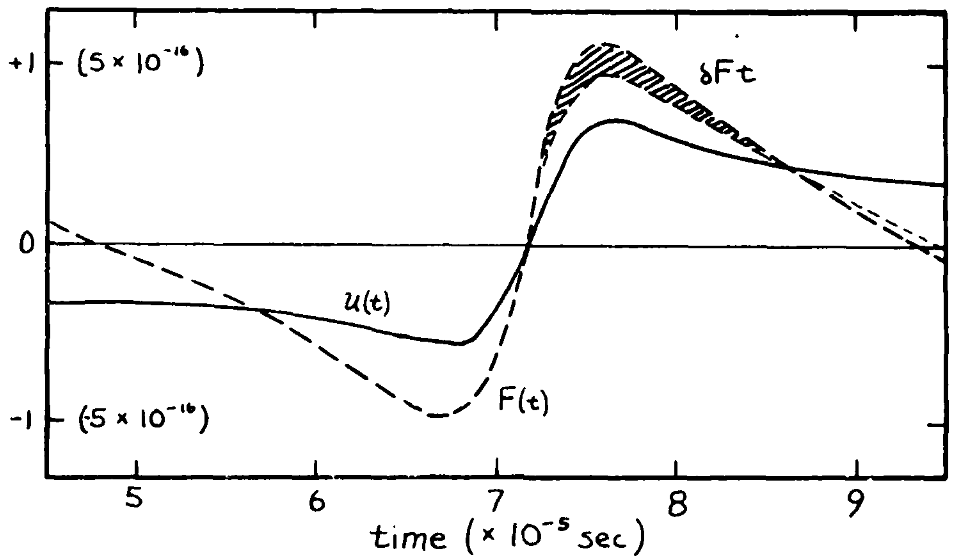


Figure 6. Bloch variable u and force (dynes) for $\Omega_o = 5A$ around a node in the standing wave. (Shaded area is the difference between the areas under the positive and negative force curves).

Figures (5) and (6) illustrate the effect of the value of u around the nodes in the standing wave on the light pressure force. In Figure (5), the net (or average) force over one cycle is negative meaning that the atom is cooled during its motion through the standing wave. In Figure (6), however, the net force is positive because of the difference between the values of u before and after the node, thus it actually increases the velocity of the atom over one period.

<u>Time-Averaged Force Reversals</u>

Figure (7) is a plot of the time-averaged radiation force calculated as a function of velocity for two different field strengths. This figure is comparable to the plot obtained by Minogin and Serimaa in Figure (1).

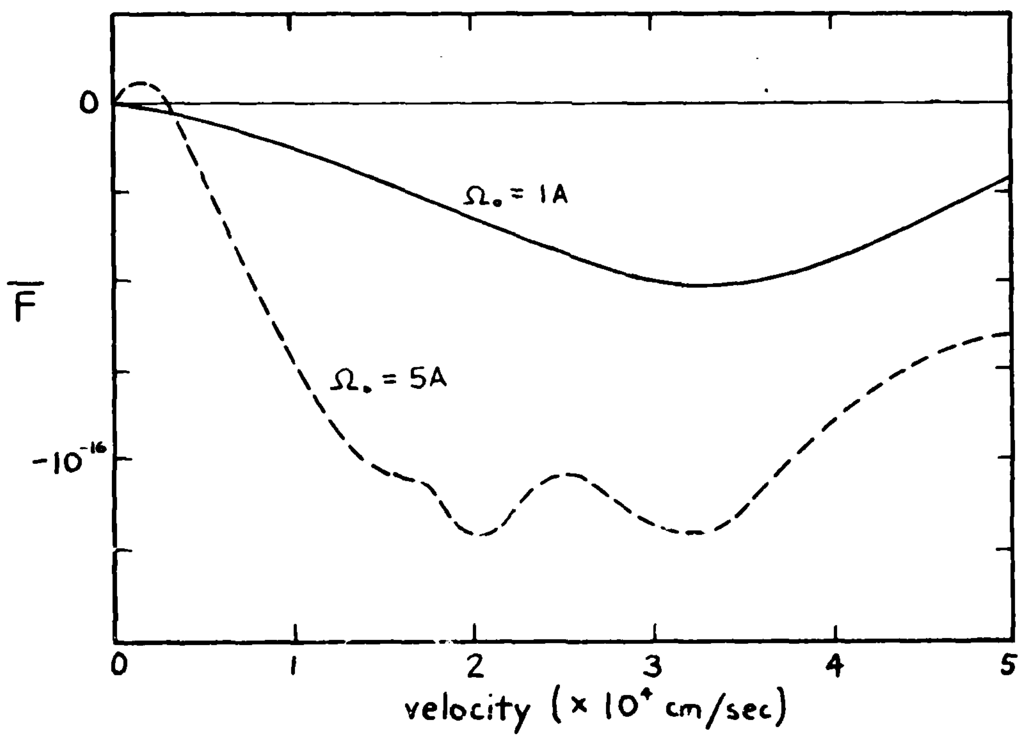


Figure 7. Time-averaged force as a function of velocity for two field strengths.

The difference between the two plots is that the detuning was $\Delta = 1A$ instead of $3A/2$.

Figure (8) shows how the radiation force at the velocity, $v = 2\times10^{3}\,\mathrm{cm/sec}$, undergoes a reversal as the field strength increases.

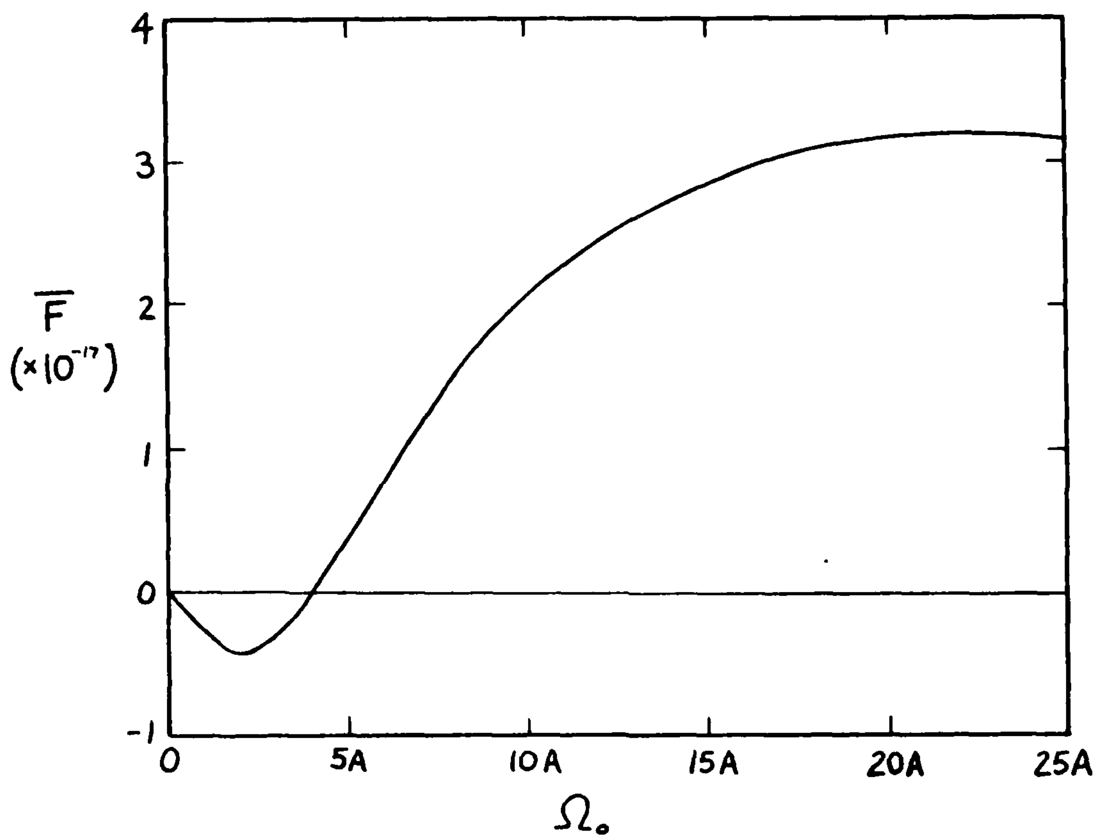


Figure 8. Time-averaged force for $v = 2\times10^{3}\,\mathrm{cm/sec}$ as a function of field strength.

Comment on the Constant Velocity Approximation

The radiation force on an atom moving through a standing light wave is given by equation (7)

$$F = -\tfrac{1}{2}\hbar k\,\Omega_0\,u\,\sin kvt \qquad (47)$$

The product of u and $\sin kvt$ is on the order of unity, so the magnitude of the force is approximately

$$|F| \sim \tfrac{1}{2}\hbar k\,\Omega_0 \tag{48}$$

The direction of this force depends on the portion of the standing wave that the atom is in. This force acts for the distance of approximately $\lambda/4$.

If the atom is moving with a constant velocity V, then the length of time that the force will be acting on the atom is $\tau = \lambda/4v$.

The change in momentum of the atom will then be

$$\delta p = m\delta v = F\tau \tag{49}$$

or

$$\delta p = \tfrac{1}{2}\hbar k\Omega_0 \frac{\lambda}{4v} = \frac{\pi\hbar\Omega_0}{4v} \tag{50}$$

The percentage change in velocity is thus given by

$$\frac{\delta v}{v} = \frac{\pi\hbar\Omega_0}{4mv^2} \tag{51}$$

or, by using approximate values,

$$\frac{\delta v}{v} \sim \frac{(10^{-27})(10^{8})}{(10^{-23})(10^{4})^2} = \frac{1}{10^4} \tag{52}$$

For the velocities considered in this study (10^3 to 10^4 cm/sec), the change in velocity over 1/4 period of the standing wave is on the order of .01%. Thus, the assumption of a constant velocity is a fairly good approximation. If,

however, the atomic velocity is of the order of 10^2 or less, or the field strength is of the order of $\Omega_0 = 10^9$ to 10^{10} , then the constant velocity assumption is of a questionable value.

Variable Velocity Program

To get an accurate picture of the atomic motion in a standing wave, the velocity of the atom cannot be assumed to be constant. It will vary according to the strength of the light pressure the atom encounters and the length of time the force is applied.

The computer program differs from the constant velocity Ehrenfest-Bloch program in that the complete Ehrenfest-Bloch equation set is used to calculate the motion of the atom. The instantaneous force is calculated at each time step, then used to calculate the change in atomic velocity. The atomic mass was taken to be $m = 4 \times 10^{-23}\,g$ which represents an atom about the weight of a sodium atom. Again, the A-coefficient and the resonant transition frequency were chosen as convenient values only, and do not represent values for real atoms.

The average light pressure force on the atom can be calculated by the average change in the velocity over one period of the standing wave.

$$\overline{F} = m \overline{\frac{\delta v}{\delta t}} \tag{53}$$

Several points of the average force were calculated this way and are plotted as triangles against the calculated curve in

Figure (9). It is noted that the values match very well for low field strengths, but differ for intense fields. As expected, the average force determined by equation (53) using the variable velocity program deviates from the curve calculated by the constant velocity program for higher field strengths.

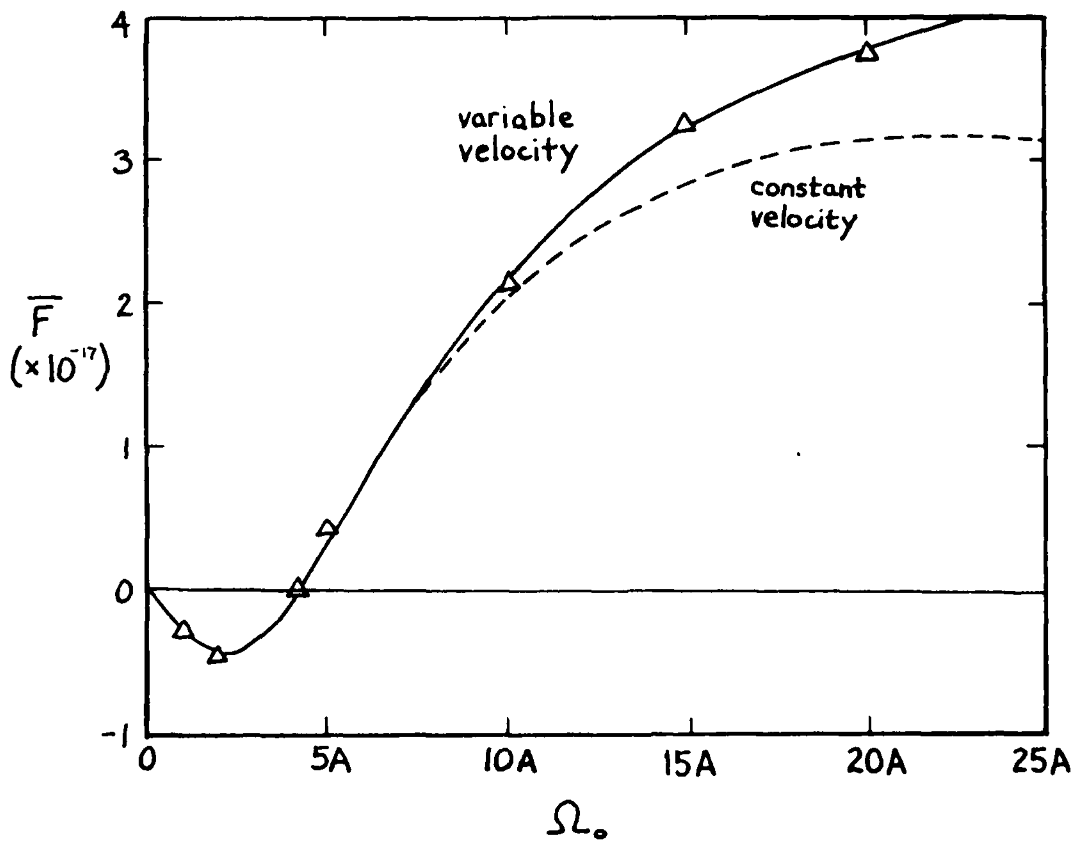


Figure 9. Comparason of time-averaged force calculated from variable velocity program and constant velocity program.

Fast Fourier Transform

A 256-point Fast Fourier Transform (FFT) program was written in Fortran 77 based on a program for a microcomputer [12]. This program was used to decompose the functions for the Bloch variable u and the radiation force from the constant velocity program, into their constituent harmonic frequencies. The time increment was adjusted to allow 256 time steps per period of the standing wave, and the output of the FFT program was a descrete harmonic frequency spectrum of the variable being studied. This program is listed in Appendix B.

Figure (10) shows the frequency spectrum for the Bloch variable u for several field strengths. As expected from studying the weak field case, the contribution by the higher harmonic terms is dependent on the intensity of the field, with the higher harmonic terms becoming negligible for relatively weak fields.

The radiation force frequency spectrum plotted in Figure (11) also shows the dependence of the higher harmonic terms on the field intensity. The force is seen to contain a set of even harmonic terms in accordance with the results of Minogin and Serimaa in equations (2). The use of this FFT program is limited, however, because of the inability to match exactly 256 steps in one period. This caused the harmonic terms to bleed over onto the adjacent harmonics. To correct for this, it would be necessary to run a 512 point or a 1024 point FFT program on the same interval.

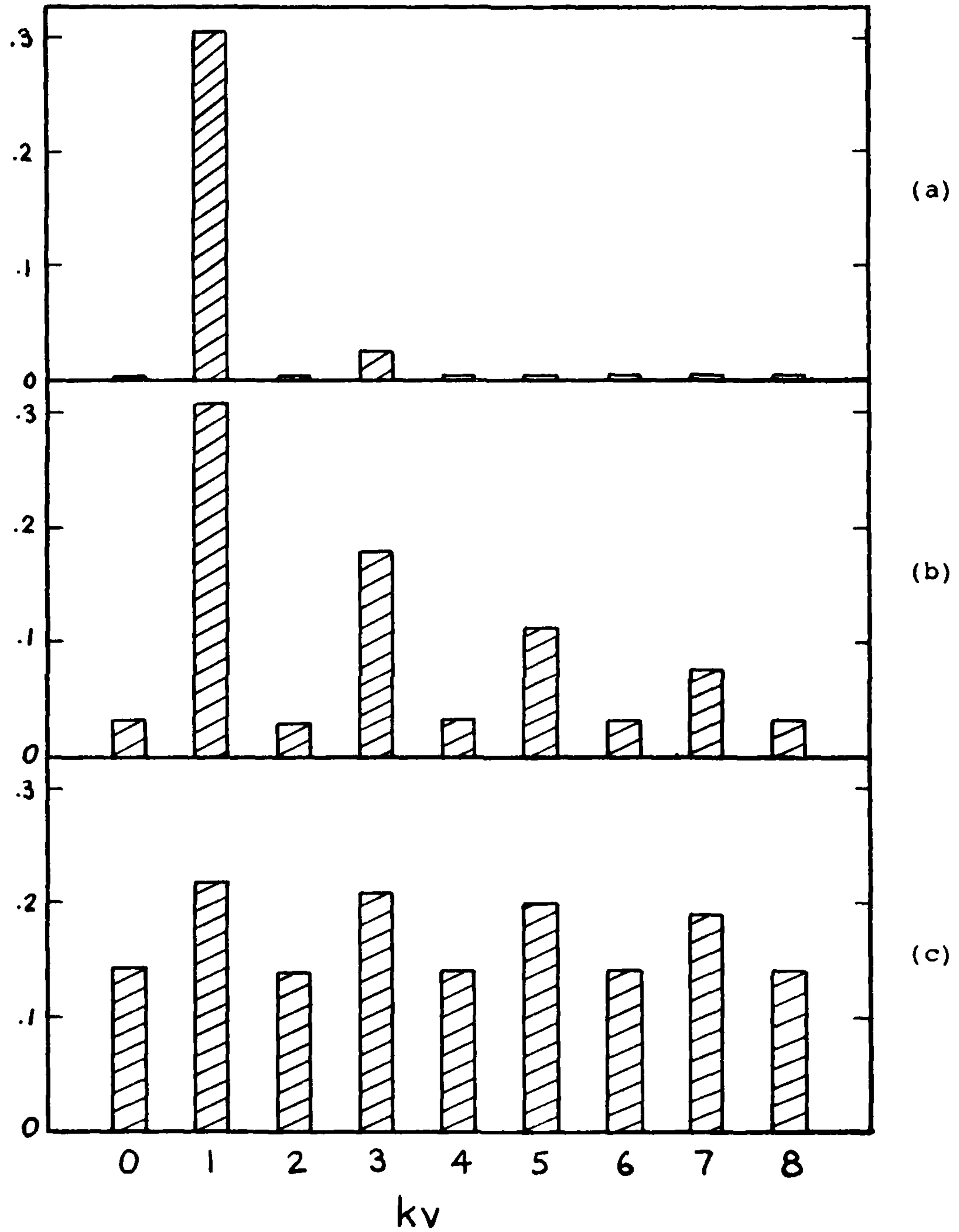


Figure 10. Descrete harmonic frequency spectrum for u calculated at: (a) $\Omega_0 = 1A$, (b) $\Omega_0 = 5A$, and (c) $\Omega_0 = 20A$.

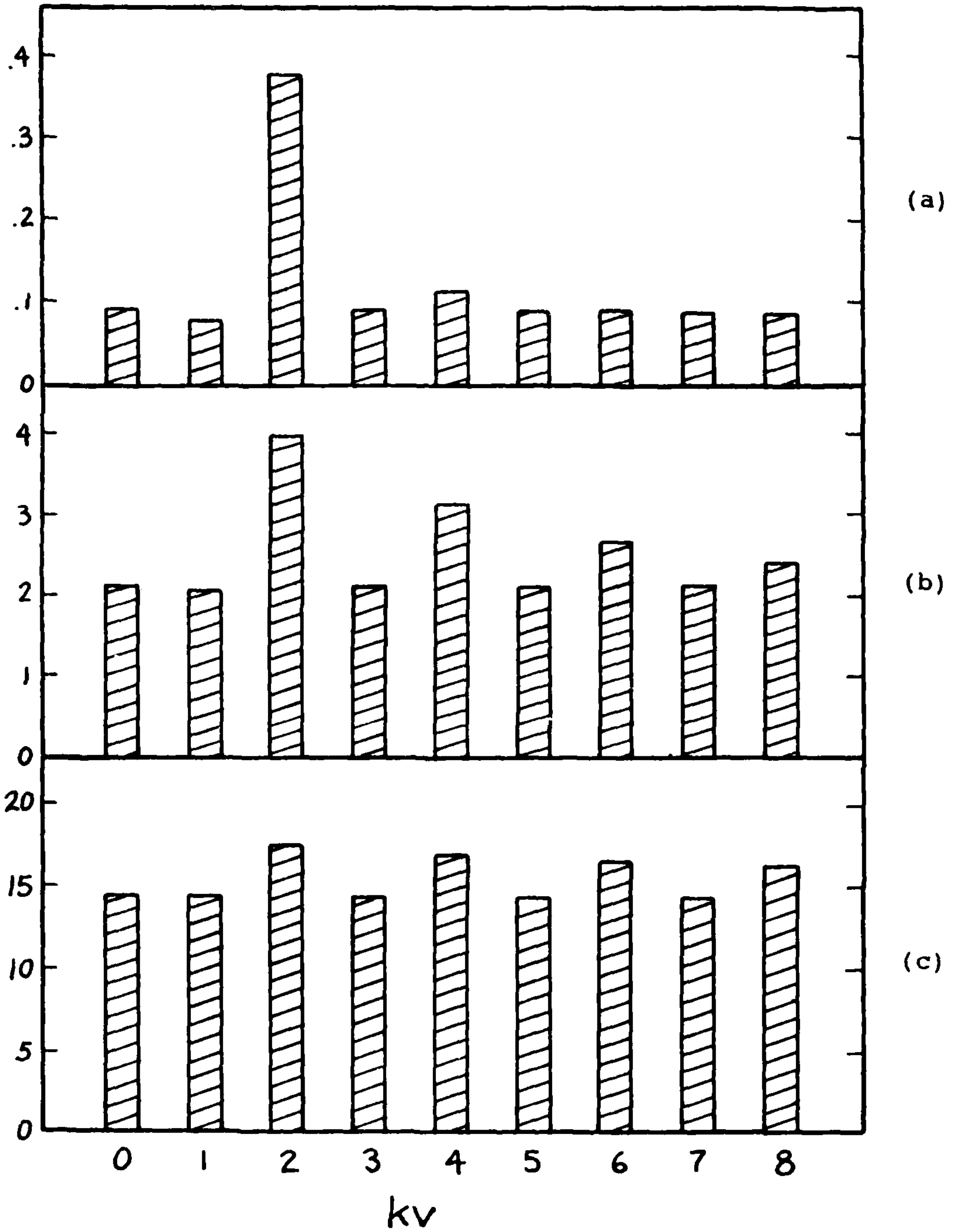


Figure 11. Descrete harmonic frequency spectrum for the radiation force calculated at: (a) $\Omega_o = 1A$, (b) $\Omega_o = 5A$, and (c) $\Omega_o = 20A$.

Conclusions and Recommendations

This study of atomic motion in a standing wave was conducted in two parts: The first, an analytical approach to solving the optical Bloch equations, and the second, a numerical approach to solve the complete Ehrenfest-Bloch equations. The analytical approach was unsuccessful in obtaining a complete solution to the Bloch equations, but several recursion formulas were obtained that can be approximated and solved for specific cases such as the weak field case. The numerical approach was successful in obtaining instantaneous values for the Bloch variables and the light pressure force. Several examples of calculations done numerically were shown to illustrate the use of the computer programs.

From the numerical calculations, it was shown that the Ehrenfest-Bloch equations indeed represent the same elements of resonance radiation pressure theory as the theories developed by Minogin, Letokhov, and Serimaa. This is because both theories are developed along the same lines from the dipole representation of the interaction between the atom and the electromagnetic wave. The difference is that Cook's theory goes a step farther to use the Bloch vector representation of the atomic density matrix to make the theory somewhat simpler and easier to visualize. The resulting Ehrenfest-Bloch theorem allows for a quick interpretation of the internal state of the atom during the

interaction with the light wave.

The Fourier series solution obtained by Minogin and Serimaa [11] can probably be obtained from the optical Bloch equations, but because of the complexity of the convergent continued fraction coefficients, its usefulness beyond some reasonable approximation is questionable. With an extremely simple computer program, the Ehrenfest-bloch equations can be solved numerically to get the same results as shown by comparing Figures (1) and (7).

Another difficulty with the solution developed by Minogin and Serimaa lies in the assumption of a constant atomic velocity. It was shown that for low to moderate field strengths, this assumption was valid, but as the field strength increased, the assumption of constant velocity became a questionable approximation. In contrast, the Ehrenfest-Bloch equations can be programmed to allow for a variable atomic velocity, a feature that would be necessary for very slow atoms and very intense fields.

One interesting feature which was found by Minogin and Serimaa and explained by Kyrölä and Stenholm was the existence of force variations including a force reversal for relatively slow atomic velocities. The explaination of an multi-photon or doppleron resonance with the two counter-propagating light waves in the standing wave makes good physical sense. But by investigating the phenomenon using the Bloch vector representation, it may be possible to develop a theory of resonance involving the classical dipole

moment of the atom and the classical electromagnetic wave. Such a resonance could very likely include some form of the Rabi flopping frequency which describes the absorption and subsequent emission of energy from the wave.

Thus, this thesis leads to several recommendations to further the study of resonance radiation. First, the Bloch representation of the atom can be studied in the standing wave interaction to try to understand and explain the resonance that causes reversals in the average light pressure force. Second, the Ehrenfest-Bloch equations can be programmed to represent an actual experimental setup and the determine the results that are expected. Third, the analytical solution can be pursued to obtain a solution similar to equations (2), but, as stated before, the use of such a solution may have limited use except to verify the theoretical development.

An area that deserves more investigation is when the time-averaged light pressure force changes sign for relatively small atomic velocities. Because the force goes to zero at three atomic velocities, $v=0$ and $v=\pm v_0$, the effect on a distribution of atomic velocities would be to separate the atoms into three groups, one that has zero velocity and two that are moving in opposite directions with the same speed. Such an experiment may be accomplished using a slow atomic beam passing perpendicularly through a large region with a standing laser wave. The transverse thermal velocities will be damped out and the resulting beam

profile whould show the three transverse velocity groups. Another experiment could involve the use of a cavity resonator in which a distribution of atoms would be irradiated, then the spectrum immediately observed with a probe laser to determine the velocity distribution of the atoms.

Through a judicious use of resonant radiation pressure, it is possible to manipulate matter on the atomic level. Such experiments can be applied to test theoretical calculations of light pressure and may eventually lead to the successful cooling and trapping of neutral atoms.

APPENDIX A: The Optical Bloch Equations

The optical Bloch equations (A-1) form a system of three equations which completely describe the internal motion of a two-level atom interacting with light in the Rotating Wave Approximation.

$$\dot{u} = -\tfrac{1}{2}Au + \Delta v \qquad \text{(A-1a)}$$

$$\dot{v} = -\Delta u - \tfrac{1}{2}Av + \Omega w \qquad \text{(A-1b)}$$

$$\dot{w} = -\Omega v - A(w+1) \qquad \text{(A-1c)}$$

$\vec{V} = (u, v, w)$ is the Bloch vector of the atom, A is the Einstein A-coefficient for spontaneous emission, Ω is the Rabi flopping frequency, and Δ is the detuning of the frequency of the incident light from the resonant transition frequency of the atom.

The Bloch equations are derived from a representation of the equation of motion of the density matrix, $\hat{\rho}$, of the atom. The density matrix is defined by

$$\hat{\rho} \equiv \sum_n P_n \, |\psi_n\rangle\langle\psi_n| \qquad \text{(A-2)}$$

where ψ_n is the wavefunction of the state n, and P_n is the probability of the atom being in that state. The equation of motion is given by the Heisenberg equation:

$$\frac{\partial\hat{\rho}}{\partial t} = \frac{1}{i\hbar}\left[\hat{H}, \hat{\rho}\right] \qquad \text{(A-3a)}$$

or in matrix form,

$$\dot{\rho}_{nm} = \frac{1}{i\hbar} \sum_k \left(H_{nk}\rho_{km} - \rho_{nk}H_{km} \right) \tag{A-3b}$$

Each state has its corresponding eigenvalue such that

$$H_{nm} = E_n \delta_{nm} \tag{A-4}$$

Equation (A-3b) thus becomes

$$\dot{\rho}_{nm} = \frac{1}{i\hbar} \sum_k \left(E_n \delta_{nk}\rho_{km} - \rho_{nk}E_k\delta_{km} \right) \tag{A-5}$$

$$\dot{\rho}_{nm} = \frac{1}{i\hbar} \left(E_n\rho_{nm} - \rho_{nm}E_m \right) \tag{A-6}$$

or

$$\dot{\rho}_{nm} = -i \left[\frac{E_n - E_m}{\hbar} \right] \rho_{nm} \tag{A-7}$$

Now, the Bohr transition frequency between two states is

$$\omega_{nm} = \frac{E_n - E_m}{\hbar} \tag{A-8}$$

thus equation (A-7) becomes

$$\dot{\rho}_{nm} = -i\omega_{nm}\rho_{nm} \tag{A-9}$$

The solution of this equation is

$$\rho_{nm}(t) = \rho_{nm}(0)\, e^{-i\omega_{nm}t} \tag{A-10}$$

In the case of $n = m$, then $\omega_{nm} = 0$, and the probability of being in state n would be time independent. This is corrected by adding the effects of spontaneous emission.

For diagonal matrix elements ($n = m$), then

equation (A-9) becomes

$$\dot{\rho}_{nn} = \sum_{p>n} A_{pn}\rho_{pp} - \sum_{q<n} A_{nq}\rho_{nn} \qquad \text{(A-11)}$$

where $A_{nm} = 4|\hat{\vec{\mu}}_{nm}|^2 \omega_{nm}^2/3\hbar c^3$. The first term in equation (A-11) represents the increase in probability of being in state n due to the decay from states p above state n, and the second term represents the decrease due to losses to lower states q .

For off diagonal matrix elements ($n \neq m$), equation (A-9) becomes

$$\dot{\rho}_{nm} = -i\omega_{nm}\rho_{nm} - \frac{1}{2}\left(\sum_{q<n} A_{nq} + \sum_{q<m} A_{mq}\right)\rho_{nm} \qquad \text{(A-12)}$$

Now, if the atom interacts with an external electric field, the Hamiltonian can be represented by

$$\hat{H} = \hat{H}_o + \hat{H}' \qquad \text{(A-13)}$$

where $\hat{H}' = -\hat{\vec{\mu}} \cdot \vec{E}(t)$. If $\vec{E}(t) = \hat{\epsilon} E \cos\omega t$ is the electric field vector for a light wave of frequency ω , then

$$\hat{H}' = -\hat{\mu} E \cos \omega t \qquad \text{(A-14)}$$

where $\hat{\mu} = \hat{\vec{\mu}} \cdot \hat{\epsilon}$ is the dipole moment operator of the atom. Equation (A-9) becomes

$$\dot{\rho}_{nm} = -i\omega_{nm}\rho_{nm} + \frac{1}{i\hbar}\sum_q \left(H'_{nq}\rho_{qm} - \rho_{nq}H'_{qm}\right) \qquad \text{(A-15)}$$

With the spontaneous emission terms, this becomes

$$\dot{\rho}_{nn} = \frac{1}{i\hbar}\sum_q \left(H'_{nq}\rho_{qm} - \rho_{nq}H'_{qm}\right) + \sum_{p>n} A_{pn}\rho_{pp} - \sum_{q<n} A_{nq}\rho_{nn} \qquad \text{(A-16a)}$$

for $n = m$, and

$$\dot{\rho}_{nm} = -i\omega_{nm}\rho_{nm} + \frac{1}{i\hbar}\sum_q \left(H'_{nq}\rho_{qm} - \rho_{nq}H'_{qm}\right) - \frac{1}{2}\left(\sum_{q<n} A_{nq} + \sum_{q<m} A_{mq}\right)\rho_{nm} \qquad \text{(A-16b)}$$

for $n \neq m$.

For a two-level atom, the density matrix is a hermitian matrix

$$\rho_{nm} = \begin{pmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{pmatrix} \qquad \text{(A-17)}$$

and the dipole moment operator matrix is

$$\langle\hat{\mu}\rangle = \begin{pmatrix} 0 & \mu \\ \mu & 0 \end{pmatrix} \qquad \text{(A-18)}$$

where μ is a real number. Equations (A-16a) and (A-16b) become

$$\dot{\rho}_{11} = \frac{1}{i\hbar}\left(-\mu E\cos\omega t\,\rho_{21} + \mu E\cos\omega t\,\rho_{12}\right) + A_{21}\rho_{22} \qquad \text{(A-19a)}$$

$$\dot{\rho}_{22} = \frac{1}{i\hbar}\left(-\mu E\cos\omega t\,\rho_{12} + \mu E\cos\omega t\,\rho_{21}\right) - A_{21}\rho_{22} \qquad \text{(A-19b)}$$

$$\dot{\rho}_{12} = -i\omega_{12} + \frac{1}{i\hbar}\left(-\mu E\cos\omega t\,\rho_{11} + \mu E\cos\omega t\,\rho_{22}\right) - \frac{1}{2}A_{21}\rho_{12} \qquad \text{(A-19c)}$$

$$\dot{\rho}_{21} = \dot{\rho}_{12}^{*} \qquad \text{(A-19d)}$$

or

$$\dot{\rho}_{11} = i\Omega \cos \omega t \left(\rho_{21} - \rho_{12}\right) + A\rho_{22} \tag{A-20a}$$

$$\dot{\rho}_{22} = i\Omega \cos \omega t \left(\rho_{12} - \rho_{21}\right) - A\rho_{22} \tag{A-20b}$$

$$\dot{\rho}_{12} = i\omega_0 + i\Omega \cos \omega t \left(\rho_{22} - \rho_{11}\right) \tag{A-20c}$$

$$\dot{\rho}_{21} = \dot{\rho}_{12}^{*} \tag{A-20d}$$

where $\Omega = \mu E/\hbar$ is the Rabi flopping frequency and $A = A_{21}$. In these equations, $\omega_0 = \omega_{21} = -\omega_{12}$ is the resonant transition frequency of the two-level atom.

Now, the density matrix of the atom can be rewritten in terms of a new matrix σ_{nm} that is a slowly varying matrix in the near-resonance case in which the dominant term (ω near ω_0) has been removed.

$$\rho_{11}(t) = \sigma_{11}(t) \tag{A-21a}$$

$$\rho_{22}(t) = \sigma_{22}(t) \tag{A-21b}$$

$$\rho_{12}(t) = \sigma_{12}(t)\, e^{i\omega t} \tag{A-21c}$$

Equations (A-20) then become

$$\dot{\sigma}_{11} = -i\Omega \cos \omega t \left(\sigma_{12} e^{i\omega t} - \sigma_{21} e^{-i\omega t}\right) + A\sigma_{22} \tag{A-22a}$$

$$\dot{\sigma}_{22} = i\Omega \cos \omega t \left(\sigma_{12} e^{i\omega t} - \sigma_{21} e^{-i\omega t}\right) - A\sigma_{22} \tag{A-22b}$$

$$\dot{\sigma}_{12} = i\Omega \cos \omega t \left(\sigma_{22} - \sigma_{11}\right) e^{-i\omega t} - i(\omega - \omega_0)\sigma_{12} - \tfrac{1}{2}A\sigma_{12} \tag{A-22c}$$

$$\dot{\sigma}_{21} = \dot{\sigma}_{12}^{*} \tag{A-22d}$$

But,

$$\cos\omega t\, e^{i\omega t} = \tfrac{1}{2}\left[e^{i\omega t} + e^{-i\omega t}\right]e^{i\omega t} = \tfrac{1}{2}\left[e^{i2\omega t} + 1\right] \tag{A-23}$$

In the Rotating Wave Approximation (RWA), the terms which vary at twice the dominant frequency are dropped since they vary rapidly and average to zero.

$$\cos\omega t\, e^{i\omega t} \approx \tfrac{1}{2} \tag{A-24}$$

Therefore, equations (A-22) become

$$\dot{\sigma}_{11} = -\tfrac{1}{2}\Omega\, i\left(\sigma_{12} - \sigma_{21}\right) + A\sigma_{22} \tag{A-25a}$$

$$\dot{\sigma}_{22} = \tfrac{1}{2}\Omega\, i\left(\sigma_{12} - \sigma_{21}\right) - A\sigma_{22} \tag{A-25b}$$

$$\dot{\sigma}_{12} = \tfrac{1}{2}\Omega\, i\left(\sigma_{22} - \sigma_{11}\right) - i\Delta\sigma_{12} - \tfrac{1}{2}A\sigma_{12}. \tag{A-25c}$$

$$\dot{\sigma}_{21} = \dot{\sigma}_{12}^{*} \tag{A-25d}$$

Now, the Bloch vector is defined by

$$u \equiv \sigma_{12} + \sigma_{21} = 2\,\mathrm{Re}\left[\sigma_{12}\right] \tag{A-26a}$$

$$v \equiv -i\left(\sigma_{12} + \sigma_{21}\right) = 2\,\mathrm{Im}\left[\sigma_{12}\right] \tag{A-26b}$$

$$w \equiv \sigma_{22} - \sigma_{11} = P_2 - P_1 \tag{A-26c}$$

where the variable w is the inversion between the two states. In the case of the two-level atom, $P_1 + P_2 = 1$.

Subtracting (A-25a) from (A-25b) provides

$$\left(\dot{\sigma}_{22}-\dot{\sigma}_{11}\right)=\Omega i\left(\sigma_{12}-\sigma_{21}\right)-A\left(\sigma_{22}+\sigma_{22}\right) \tag{A-27}$$

$$\left(\dot{\sigma}_{22}-\dot{\sigma}_{11}\right)=-\Omega\left[-i\left(\sigma_{12}-\sigma_{21}\right)\right]-A\left[\left(\sigma_{22}-\sigma_{11}\right)+\left(\sigma_{22}+\sigma_{11}\right)\right] \tag{A-28}$$

Adding (A-25c) to (A-25d) provides

$$\left(\dot{\sigma}_{12}+\dot{\sigma}_{21}\right)=-\tfrac{1}{2}A\sigma_{12}-\tfrac{1}{2}A\sigma_{21}-i\Delta\sigma_{12}+i\Delta\sigma_{21} \tag{A-29}$$

$$\left(\dot{\sigma}_{12}+\dot{\sigma}_{21}\right)=-\tfrac{1}{2}A\left(\sigma_{12}+\sigma_{21}\right)+\Delta\left[-i\left(\sigma\ \ -\sigma_{21}\right)\right] \tag{A-30}$$

And subtracting (A-25d) from (A-25c) provides

$$\begin{aligned}\left(\dot{\sigma}_{12}-\dot{\sigma}_{21}\right)= {} & i\Delta\sigma_{12}-i\Delta\sigma_{21}-\tfrac{1}{2}A\sigma_{12}+\tfrac{1}{2}A\sigma_{21}\\ & +\Omega i\left(\sigma_{22}-\sigma_{11}\right)\end{aligned} \tag{A-31}$$

$$\begin{aligned}-i\left(\dot{\sigma}_{12}-\dot{\sigma}_{21}\right)= {} & -\Delta\left(\sigma_{12}+\sigma_{21}\right)-\tfrac{1}{2}A\left[-i\left(\sigma_{12}-\sigma_{21}\right)\right]\\ & +\Omega\left(\sigma_{22}-\sigma_{11}\right)\end{aligned} \tag{A-32}$$

And finally, with the appropriate substitutions, the familiar form of the optical Bloch equations is derived

$$\dot{u}=-\tfrac{1}{2}Au+\Delta v \tag{A-33a}$$

$$\dot{v}=-\Delta u-\tfrac{1}{2}Av+\Omega w \tag{A-33b}$$

$$\dot{w}=-\Omega v-A(w+1) \tag{A-33c}$$

The Bloch vector provides a useful description of the two-level atom in the RWA. u and v are components

of the dipole moment, μ , such that

$$\mu = u + iv \tag{A-34}$$

u is the in-phase or dispersive component, while v is the in-quadrature or absorptive component [1].

APPENDIX B: Computer Source Code

Constant Velocity Program

```
C   BLOCH SOLVER
C
C   THIS PROGRAM CALCULATES THE BLOCH VECTOR COMPONENTS AND
C   THE INSTANTANEOUS LIGHT PRESSURE FORCE ON AN ATOM USING
C   THE EHRENFEST-BLOCH EQUATIONS WITH A CONSTANT VELOCITY
C   ASSUMPTION.
C
        CHARACTER*8 FILE1
        REAL A,C,H,K,T,DEL,F0,F1,OM
        REAL U0,V0,W0,FRC,V
        COMMON K,V,DEL,OM
C
        C=2.99792458E10
        F0=1.0E14
        DEL=-1.0E8
        F1=F0+DEL
        K=F1/C
C
C   THE ATOM STARTS IN THE GROUND STATE (0,0,-1)
C
        T=0.0
        U0=0.0
        V0=0.0
        W0=-1.0
C
        WRITE(6,5)
5       FORMAT(/' ENTER OUTPUT DATAFILE NAME')
        READ(6,10) FILE1
10      FORMAT(A6)
C
        OPEN(UNIT=10,FILE=FILE1,STATUS='NEW',FORM='FORMATTED')
C
        WRITE(6,15)
15      FORMAT(///' INPUT OMEGA')
        READ(6,20) OM
20      FORMAT(E6.4)
        WRITE(6,25)
25      FORMAT(' INPUT ATOMIC VELOCITY')
        READ(6,20) V
C
C   STORE THE RUN INFORMATION IN FILE 10.
C
40      WRITE(10,50) OM,V,F0,F1,DEL
50      FORMAT(////'                    OMEGA = ',E10.5/
      + '        ATOMIC VELOCITY = ',E10.5/' TRANSITION ',
      + 'FREQUENCY = ',E10.5/'     LASER FREQUENCY = ',
      + E10.5/'                  DELTA = ',E11.5//)
C
```

```
      WRITE(10,60)
60    FORMAT(6X,'TIME',9X,'U(t)',10X,'V(t)',10X,'W(t)',10X,
     + 'FORCE'/'-------------------------------------------',
     + '-----------------------------')
      WRITE(10,110) T,U0,V0,W0,FRC
C
C  CALCULATE THE BLOCH VECTOR UNTIL THE STOP TIME AND STORE
C  THE VALUES IN FILE 10.
C
100   CALL BLOCH(T,U0,V0,W0,FRC)
      WRITE(10,110) T,U0,V0,W0,FRC
110   FORMAT(' ',E10.3,4(2X,E12.5))
      IF(T.LE.5.0E-6) GOTO 100
      STOP
      END
C
C
      SUBROUTINE BLOCH(T,U0,V0,W0,FRC)
      REAL T,U0,V0,W0,FRC
      REAL K,V,DEL,OM
      COMMON K,V,DEL,OM
C
      A=1.0E8
      H=1.0E-11
      HBAR=1.054592E-27
C
      DO 200 I=1,500
C
      OMEGA=OM*COS(K*V*T)
C
C  OPTICAL BLOCH EQUATIONS
C
      DU=-.5*A*U0+DEL*V0
      DV=-DEL*U0-.5*A*V0+OMEGA*W0
      DW=-OMEGA*V0-A*W0-A
C
C  CALCULATE THE BLOCH VECTOR AFTER THE TIME STEP H.
C
      T=T+H
      U0=DU*H+U0
      V0=DV*H+V0
      W0=DW*H+W0
C
200   CONTINUE
C
C  EQUATION OF MOTION FROM EHRENFEST-BLOCH EQUATIONS
C
      FRC=-.5*K*HBAR*U0*OM*SIN(K*V*T)
C
      RETURN
      END
```

Variable Velocity Program

```
C   EHRENFEST-BLOCH SOLVER
C
C   THIS PROGRAM CALCULATES THE BLOCH VECTOR COMPONENTS AND
C   THE INSTANTANEOUS LIGHT PRESSURE FORCE ON AN ATOM USING
C   THE EHRENFEST-BLOCH EQUATIONS WITH THE VELOCITY ALLOWED
C   TO VARY.
C
      CHARACTER*8 FILE1
      REAL A,C,H,K,T,DEL,F0,F1,OM,MASS
      REAL U0,V0,W0,FRC,X,V
      COMMON K,DEL,OM,MASS
C
      MASS=4.0E-23
      C=2.99792458E10
      F0=1.0E14
      DEL=-1.0E8
      F1=F0+DEL
      K=F1/C
C
C   THE ATOM STARTS IN THE GROUND STATE (0,0,-1)
C
      T=0.0
      X=0.0
      U0=0.0
      V0=0.0
      W0=-1.0
C
      WRITE(6,5)
5     FORMAT(/' ENTER OUTPUT DATAFILE NAME')
      READ(6,10) FILE1
10    FORMAT(A6)
C
      OPEN(UNIT=10,FILE=FILE1,STATUS='NEW',FORM='FORMATTED')
C
      WRITE(6,15)
15    FORMAT(/' INPUT OMEGA')
      READ(6,20) OM
20    FORMAT(E6.4)
      WRITE(6,25)
25    FORMAT(' INPUT INITIAL ATOMIC VELOCITY')
      READ(6,20) V
C
C   STORE THE RUN INFORMATION IN FILE 10.
C
40    WRITE(10,50) OM,V,F0,F1,DEL
50    FORMAT(////'                          OMEGA = ',E10.5/
     + ' INITIAL ATOMIC VELOCITY = ',E10.5/' TRANSITION ',
     + 'FREQUENCY = ',E10.5/'        LASER FREQUENCY = ',
     + E10.5/'                      DELTA = ',E11.5//)
C
```

```
      WRITE(10,60)
60    FORMAT(6X,'TIME',9X,'U(t)',10X,'V(t)',10X,'W(t)',9X,
     + 'FORCE',9X,'X(t)',9X,'VEL(t)'/'--------------------',
     + '----------------------------------------------------',
     + '--------------------------')
      WRITE(10,110) T,U0,V0,W0,FRC,X,V
C
C  CALCULATE THE BLOCH VECTOR UNTIL THE STOP TIME AND STORE
C  THE VALUES IN FILE 10.
C
100   CALL EHREN(T,U0,V0,W0,X,V,FRC)
      WRITE(10,110) T,U0,V0,W0,FRC,X,V
110   FORMAT(' ',E10.3,5(2X,E12.5) ,2X,E15.8)
      IF(T.LE.5.0E-6) GOTO 100
      STOP
      END
C
C
      SUBROUTINE EHREN(T,U0,V0,W0,X,V,FRC)
      REAL T,U0,V0,W0,FRC,X,MASS
      REAL K,V,DEL,OM
      COMMON K,DEL,OM,MASS
C
      A=1.0E8
      H=1.0E-11
      HBAR=1.054592E-27
C
      DO 200 I=1,500
C
      OMEGA=OM*COS(K*X)
C
C  OPTICAL BLOCH EQUATIONS
C
      DU=-.5*A*U0+DEL*V0
      DV=-DEL*U0-.5*A*V0+OMEGA*W0
      DW=-OMEGA*V0-A*W0-A
C
C  CALCULATE THE BLOCH VECTOR AFTER THE TIME STEP H.
C
      T=T+H
      U0=DU*H+U0
      V0=DV*H+V0
      W0=DW*H+W0
      X=X+V*H
C
C  EQUATION OF MOTION FROM EHRENFEST-BLOCH EQUATIONS
C
      FRC=-.5*HBAR*K*U0*OM*SIN(K*X)
      V=V+FRC*H/MASS
C
200   CONTINUE
C
      RETURN
      END
```

Fast Fourier Transform Program

```
C   PROGRAM FASTRAN
C
C   A 256-point FAST FOURIER TRANSFORM program adapted from
C   a program which appeared in BYTE Magazine in December
C   1978.
C
C
      CHARACTER*8 FILE1,FILE2
      REAL X1(256),X2(256)
      REAL U0(256),V0(256),W0(256),FRC(256)
      INTEGER X,Y,U
      COMMON L,N
      N=256
      L=8
      PI=3.141592654
C
C   INPUT TIME-DEPENDENT FUNCTION
C
      WRITE(6,5)
5     FORMAT(/' ENTER INPUT DATAFILE NAME')
      READ(6,6) FILE1
6     FORMAT(A6)
      WRITE(6,7)
7     FORMAT(/' ENTER OUTPUT DATAFILE NAME')
      READ(6,6) FILE2
C
      WRITE(6,10)
10    FORMAT(/' ENTER STARTING LINE NUMBER')
      READ(6,15) INDO
15    FORMAT(I3)
C
      OPEN(UNIT=10,FILE=FILE1,STATUS='OLD',FORM='FORMATTED')
      OPEN(UNIT=11,FILE=FILE2,STATUS='NEW',FORM='FORMATTED')
C
      DO 30 I=1,INDO-1
      READ(10,20) XXX
20    FORMAT(A1)
30    CONTINUE
C
      DO 50 I=0,256
      READ(10,40) U0(I),V0(I),W0(I),FRC(I)
40    FORMAT(T14,E12.5,T28,E12.5,T42,E12.5,T56,E12.5)
50    CONTINUE
C
55    WRITE(6,60)
60    FORMAT(/' WHICH VARIABLE DO YOU WANT? (1 - 4)')
      READ(6,70) IVAR
70    FORMAT(I1)
C
      GOTO(75,80,85,90) IVAR
C
```

```
C
75    DO 76 I=0,N
      X1(I)=U0(I)
76    CONTINUE
      GOTO 99
C
80    DO 81 I=0,N
      X1(I)=V0(I)
81    CONTINUE
      GOTO 99
C
85    DO 86 I=0,N
      X1(I)=W0(I)
86    CONTINUE
      GOTO 99
C
90    DO 91 I=0,N
      X1(I)=FRC(I)*.1E17
91    CONTINUE
C
99    CONTINUE
C
C  SCALE INPUT TIME FUNCTION
C
      DO 100 I=0,N-1
      X1(I)=X1(I)/N
100   CONTINUE
C
C  FFT IN-PLACE ALGORITHM
C
      I1=N/2
      I2=1
      V=2*PI/N
C
      DO 400 I=1,L
      I3=0
      I4=I1
C
      DO 300 J=1,I2
      X=INT(I3/I1)
      CALL SCRAM(X,Y)
      I5=Y
      Z1=COS(V*I5)
      Z2=-SIN(V*I5)
C
      DO 200 K=I3,I4-1
      A1=X1(K)
      A2=X2(K)
      B1=Z1*X1(K+I1)-Z2*X2(K+I1)
      B2=Z2*X1(K+I1)+Z1*X2(K+I1)
      X1(K)=A1+B1
      X2(K)=A2+B2
      X1(K+I1)=A1-B1
      X2(K+I1)=A2-B2
```

```
C
      I3=I3+2*I1
      I4=I4+2*I1
300   CONTINUE
C
      I1=I1/2
      I2=2*I2
400   CONTINUE
C
C  OUTPUT TABLE OF VALUES
C
      U=0
C
      IF(IVAR.EQ.1) WRITE(11,410)
      IF(IVAR.EQ.2) WRITE(11,420)
      IF(IVAR.EQ.3) WRITE(11,430)
      IF(IVAR.EQ.4) WRITE(11,440)
410   FORMAT(///' FFT for U(t)')
420   FORMAT(///' FFT for V(t)')
430   FORMAT(///' FFT for W(t)')
440   FORMAT(///' FFT for FRC(t)')
C
      WRITE(11,450)
450   FORMAT(/' HARMONIC    REAL    IMAGINARY    MAGNITUDE')
      WRITE(11,460)
460   FORMAT(' -------------------------------------------')
C
500 X=U
      CALL SCRAM(X,Y)
      X3=SQRT(X1(Y)*X1(Y)+X2(Y)*X2(Y))
      WRITE(11,550) U,X1(Y),X2(Y),X3
550   FORMAT('  ',I4,3(1X,F10.6))
      U=U+1
      IF(U.LE.N/2) GOTO 500
C
      WRITE(6,600)
600   FORMAT(/'CALCULATE FFT FOR ANOTHER VARIABLE?')
      READ(6,650) YES
650   FORMAT(A1)
      IF(YES.EQ.'Y') GOTO 55
C
      CLOSE(10)
      CLOSE(11)
C
      STOP
      END
C
C
C  SUBROUTINE SCRAMBLER
C
      SUBROUTINE SCRAM(X,Y)
      INTEGER X,Y
      COMMON L,N
```

```
C
      Y=0
      N1=N
C
      DO 100 I=1,L
      N1=N1/2
      IF(X.LT.N1) GOTO 100
      Y=Y+2**(I-1)
      X=X-N1
100   CONTINUE
      RETURN
      END
```

Bibliography

1. Allen, L. and J.H. Eberly Optical Resonance & Two-Level Atoms. New York: John Wiley & Sons, Inc., 1975.

2. Ashkin, A. and J.D. Gordon "Cooling and trapping of atoms by resonance radiation pressure," Optics Letters 4(6): 161-163 (1979).

3. Cook, R.J. "Atomic motion in resonant radiation: An application of Ehrenfest's theorem," Physical Review A 20(1): 224-228 (July 1979).

4. Cook, R.J. "Directions in the Theory of Resonance-radiation Pressure," Comments on Atomic and Molecular Physics 10(6): 267-277 (1981).

5. Cook, R.J. "Theory of resonance-radiation pressure," Physical Review A 22(3): 1078-1098 (September 1980).

6. Hänsch, T.W. and A.L. Schawlow "Cooling of Gases by Laser Radiation," Optics Communications 13(1): 68-69 (1975).

7. Kazantsev, A.P. "Resonance light pressure," Soviet Physics Usp. 21(1): 58-76 (Jan 1978)

8. Kyrölä, E. and S. Stenholm "Velocity tuned resonances as multi-doppleron processes," Optics Communications 22(2): 123-126 (August 1977).

9. Letokhov, V.S. and V.G. Minogin "Cooling, trapping, and storage of atoms by resonant laser fields," Journal of the Optical Society of America 69(3): 413-419 (March 1979).

10. Letokhov, V.S. and V.G. Minogin "Laser Radiation Pressure on Free Atoms," Physics Reports (Review Section of Physics Letters) 73(1): (1981).

11. Minogin, V.G. and O.T. Serimaa "Resonant light pressure forces in a strong standing laser wave," Optics Communications 30(3): 373-379 (September 1979).

12. Stanley, William D., and Steven J. Peterson "Fast Fourier Transforms on Your Home Computer," Byte: 14-25 (December 1978).

13. Stenholm, S., V.G. Minogin and V.S. Letokhov "Resonant light pressure due to a strong standing wave," Optics Communications 25(1): 107-110 (April 1978).

VITA

Joseph Edward Scott was born on 16 August 1958 in Springfield, Missouri. He graduated from high school in Fair Grove, Missouri in 1976 and attended the University of Missouri--Columbia where he received the degree of Bachelor of Science in Physics on 10 May 1980. Upon graduation, he was commissioned in the USAF as a Distinguished Graduate of the AFROTC. Assigned to the Air Force Wright Aeronautical Laboratories at Wright-Patterson AFB, Ohio, he worked in the Avionics Laboratory, Electronic Warfare Division, Active Electronic Countermeasures Branch until entering the School of Engineering, Air Force Institute of Technology, in May 1983.

Permanent address: Route 1, Box 182
Elkland, Missouri 65644

UNCLASSIFIED
SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

1a. REPORT SECURITY CLASSIFICATION
UNCLASSIFIED

1b. RESTRICTIVE MARKINGS

2a. SECURITY CLASSIFICATION AUTHORITY

2b. DECLASSIFICATION/DOWNGRADING SCHEDULE

3. DISTRIBUTION/AVAILABILITY OF REPORT
Approved for public release;
distribution unlimited.

4. PERFORMING ORGANIZATION REPORT NUMBER(S)
AFIT/GEP/ENP/84D-9

5. MONITORING ORGANIZATION REPORT NUMBER(S)

6a. NAME OF PERFORMING ORGANIZATION
School of Engineering

6b. OFFICE SYMBOL *(If applicable)*
AFIT/ENP

7a. NAME OF MONITORING ORGANIZATION

6c. ADDRESS *(City, State and ZIP Code)*
Air Force Institute of Technology
Wright-Patterson AFB, Ohio 45433

7b. ADDRESS *(City, State and ZIP Code)*

8a. NAME OF FUNDING/SPONSORING ORGANIZATION

8b. OFFICE SYMBOL *(If applicable)*

9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER

8c. ADDRESS *(City, State and ZIP Code)*

10. SOURCE OF FUNDING NOS.

PROGRAM ELEMENT NO.	PROJECT NO.	TASK NO.	WORK UNIT NO.

11. TITLE *(Include Security Classification)*
ATOMIC MOTION IN A STANDING WAVE

12. PERSONAL AUTHOR(S)
Joseph E. Scott, B.S., Captain, USAF

13a. TYPE OF REPORT
MS Thesis

13b. TIME COVERED
FROM ______ TO ______

14. DATE OF REPORT *(Yr., Mo., Day)*
1984 December

15. PAGE COUNT
66

16. SUPPLEMENTARY NOTATION

17. COSATI CODES

FIELD	GROUP	SUB. GR.
20		

18. SUBJECT TERMS *(Continue on reverse if necessary and identify by block number)*
Physics

Approved for public release: IAW AFR 190-1.
LYNN E. WOLAVER 16 JAN 86
Dean for Research and Professional Development
Air Force Institute of Technology (ATC)
Wright-Patterson AFB OH 45433

19. ABSTRACT *(Continue on reverse if necessary and identify by block number)*

Thesis Chairman: Richard J. Cook, Major, USAF

Abstract: The motion of an atom in an intense resonant standing laser wave is studied using the Ehrenfest-Bloch equations. Two approaches are taken to determine the radiation or light pressure force on the atom. The first, an attempt to find a Fourier series solution to the optical Bloch equations failed, but Fourier coefficient recursion formulas were found. The second, a numerical calculation of the Bloch variables and the radiation force was successful in obtaining plots of the instantaneous values. The effect of assuming constant atomic velocity is studied, and Fast Fourier Transform descrete harmonic frequency spectra of one of the Bloch variables and the light pressure force are obtained. Finally, the Ehrenfest-Bloch equations are compared with other theories of resonant radiation pressure.

20. DISTRIBUTION/AVAILABILITY OF ABSTRACT
UNCLASSIFIED/UNLIMITED ☒ SAME AS RPT. ☐ DTIC USERS ☐

21. ABSTRACT SECURITY CLASSIFICATION
UNCLASSIFIED

22a. NAME OF RESPONSIBLE INDIVIDUAL
Richard J. Cook, Major, USAF

22b. TELEPHONE NUMBER *(Include Area Code)*
513-255-2012

22c. OFFICE SYMBOL
AFIT/ENP

DD FORM 1473, 83 APR   EDITION OF 1 JAN 73 IS OBSOLETE.

UNCLASSIFIED
SECURITY CLASSIFICATION OF THIS PAGE